UNIT - I

ELECTROSTATICS-1

Learning objectives:

- To introduce the students to basic concepts of electric field Coulomb's Law
- To introduce electric field intensity and its calculations for different charge distributions.

Syllabus:

Introduction of Electromagnetic fields - Introduction of vector analysis - vector identies - divergence and stokes theorems - coordinate systems- Introduction to Electrostatic fields – Coulombs law - problems on Coulombs law - Force due to multiple charges - problems on multiple charges - Electric field intensity due to a line, Ring and a surface charge

Learning outcomes:

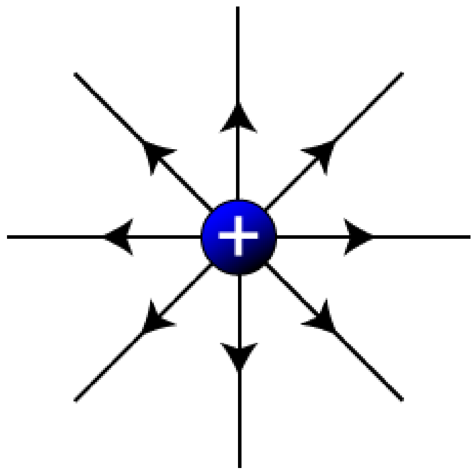
Students will be able to

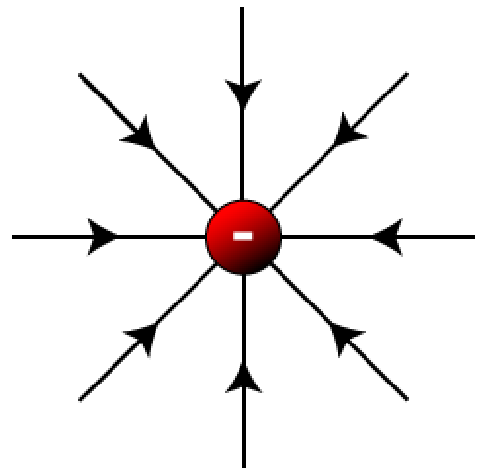
- Determine the force between two point charges.
- Define electric field intensity or electric field strength (E) and derive expression for electric field for line charge, circular ring and charged disc.

Learning Material

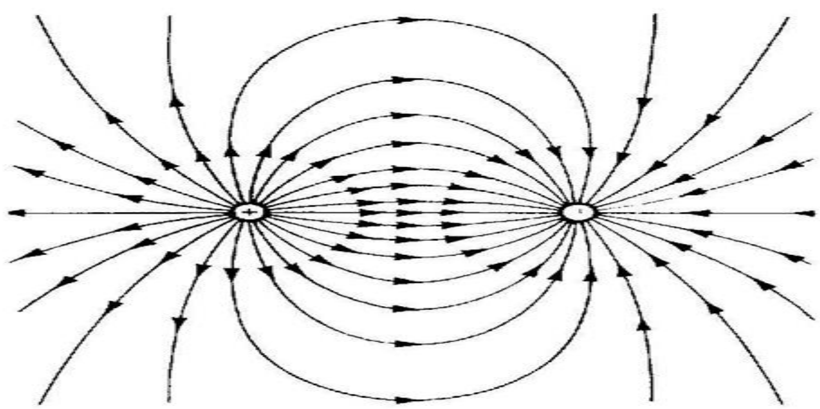
Charges at rest produce Static Electric Field or Electrostatic field.

Field: It is the existing space in particular area due to some elements.





Electric field due to isolated positive charge   Electric field due to isolated negative charge



The lines of force due to a pair of charges, one positive and the other negative

Coulomb's Law

Coulomb states that the force between two point charges separated in a vacuum or free space by a distance which is large compared to their size is

(i) proportional to magnitude of each charge
(ii) inversely proportional to the square of the distance between them
(iii) directed along the line joining the charges
(iv) it should be depend up on the medium also.

Let Q_1 and Q_2 be the two point charges separated by a distance $|\overrightarrow{R_{12}}|$ and F_2 be the force experienced by Q_2 due to Q_1

$$|F_2| \; \alpha \; \frac{Q_1 Q_2}{|\overrightarrow{R_{12}}|^2}$$

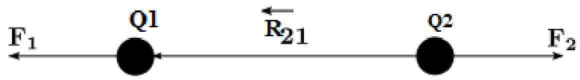


Fig. 1.1 If Q_1 and Q_2 have like signs the vector force $\boldsymbol{F_2}$ on Q_2 is in the same direction as $\overrightarrow{R_{12}}$

$$|F_2| = k \frac{Q_1 Q_2}{|\overrightarrow{R_{12}}|^2} \tag{1.1}$$

Where k is the proportionality constant. $k = 1/4\pi\varepsilon$ in SI units. The constant ε is known as the *permittivity of medium* (in farads per meter)

Where $\varepsilon = \varepsilon_0 \varepsilon_r$

ε_0 =absolute permittivity of free space = $8.854\times10^{-12} \approx \frac{10^{-9}}{36\pi}$ F/m

ε_r= Relative permittivity of medium

=1 for air or free space

$k = 1/(4\pi\varepsilon_o) = 9 \times 10^9$

Thus Eq. (1.1) becomes

$$|F_2| = \frac{1}{4\pi\varepsilon} \frac{Q_1 Q_2}{|\overrightarrow{R_{12}}|^2} \tag{1.2}$$

If point charges Q_1 and Q_2 are located at points, then the force F_2 on Q_2 due to Q_1, shown in Figure 1.1, is given by

$\overrightarrow{F_2} = |F_2| * \overrightarrow{a_{\boldsymbol{R_{12}}}}$

$$\overrightarrow{F_2} = \frac{1}{4\pi\varepsilon} \frac{Q_1 Q_2}{|\overrightarrow{R_{12}}|^2} \overrightarrow{a_{\boldsymbol{R_{12}}}} \tag{1.3}$$

Where

$$\overrightarrow{a_{\boldsymbol{R_{12}}}} = \frac{\overrightarrow{R_{12}}}{|\overrightarrow{R_{12}}|} = unit\ vector\ directed\ from\ Q_1 to\ Q_2 \tag{1.4}$$

By substituting eq. (1.4) into eq. (1.3), we may write eq. (1.3) as

$$\overrightarrow{F_2} = \frac{1}{4\pi\varepsilon}\frac{Q_1 Q_2}{\overrightarrow{|R_{12}|^3}}\overrightarrow{R_{12}} \qquad N \tag{1.5}$$

In air or free space $\overrightarrow{F_2} = \frac{1}{4\pi\varepsilon_0}\frac{Q_1 Q_2}{\overrightarrow{|R_{12}|^3}}\overrightarrow{R_{12}} = 9 \times 10^9 \frac{Q_1 Q_2}{\overrightarrow{|R_{12}|^3}}\overrightarrow{R_{12}}$ N

Similarly the force F_1 on Q_1 due to Q_2 is given by

$$\overrightarrow{F_1} = \frac{1}{4\pi\varepsilon}\frac{Q_1 Q_2}{\overrightarrow{|R_{21}|^3}}\overrightarrow{R_{21}} \qquad N$$

$$\overrightarrow{F_1} = -\frac{1}{4\pi\varepsilon}\frac{Q_1 Q_2}{\overrightarrow{|R_{12}|^3}}\overrightarrow{R_{12}} \quad = -\overrightarrow{F_2}$$

$$\overrightarrow{F_1} = -\overrightarrow{F_2} \tag{1.6}$$

Limitations of Coulombs law:

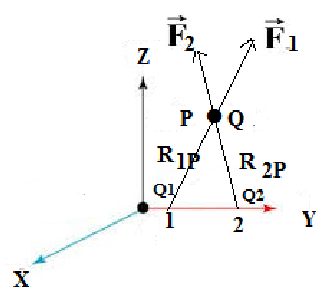
1. The coulombs law is only applicable for point charges.
2. They should be stationary with respect to each other.
3. Force between two point charges can be determined in only single medium.

Force due to 'N' no. of charges:

If we have more than two point charges, we can use *the principle of superposition* to determine the force on a particular charge. The principle states that if there are 'n'charges Q_1, Q_2, Q_3.................... $Q_n$located, respectively, at points with position vectors$\boldsymbol{R_{1p}}, \boldsymbol{R_{2p}}, \boldsymbol{R_{3p}} \ldots\ldots\ldots\ldots\ldots \boldsymbol{R_{np}}$, the resultant force $\boldsymbol{F}$ on a charge Qlocated at point(p)is the vector sum of the forces exerted on Qby each of the charges Q_1, Q_2, Q_3.................... Q_n.Hence:

$$\boldsymbol{F} = \overrightarrow{F_1} + \overrightarrow{F_2} + \cdots + \overrightarrow{F_n}$$

$$\boldsymbol{F} = \frac{1}{4\pi\varepsilon_o}\frac{QQ_1}{\overrightarrow{|R_{1p}|^3}}\overrightarrow{R_{1p}} + \frac{1}{4\pi\varepsilon_o}\frac{QQ_2}{\overrightarrow{|R_{2p}|^3}}\overrightarrow{R_{2p}} + \cdots + \frac{1}{4\pi\varepsilon_o}\frac{QQ_n}{\overrightarrow{|R_{np}|^3}}\overrightarrow{R_{np}} \tag{1.7}$$



$$\boldsymbol{F} = \frac{Q}{4\pi\varepsilon_o}\sum_{i=1}^{N}\frac{Q_i}{\overrightarrow{|R_{ip}|^3}}\overrightarrow{R_{ip}} \tag{1.8}$$

$$\boldsymbol{F} = 9 \times 10^9\, Q \sum_{i=1}^{N}\frac{Q_i}{\overrightarrow{|R_{ip}|^3}}\overrightarrow{R_{ip}} \tag{1.9}$$

Electric Field Intensity or Electric Field Strength (EFI):

It is the force per unit charge.

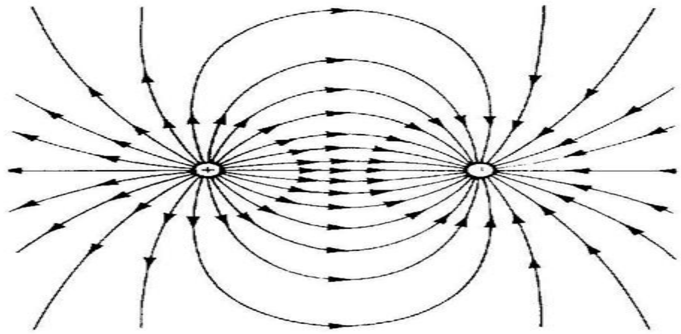


Fig 1.2 The lines of force due to a pair of charges, one positive and the other negative

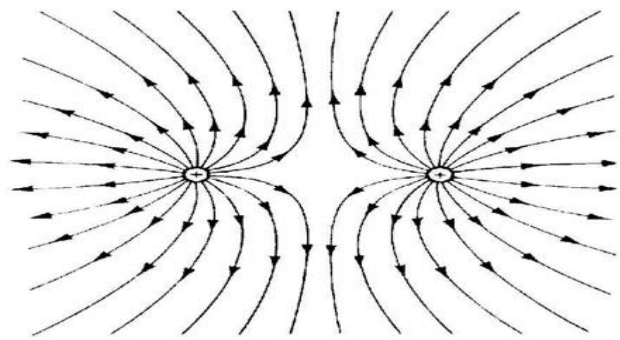
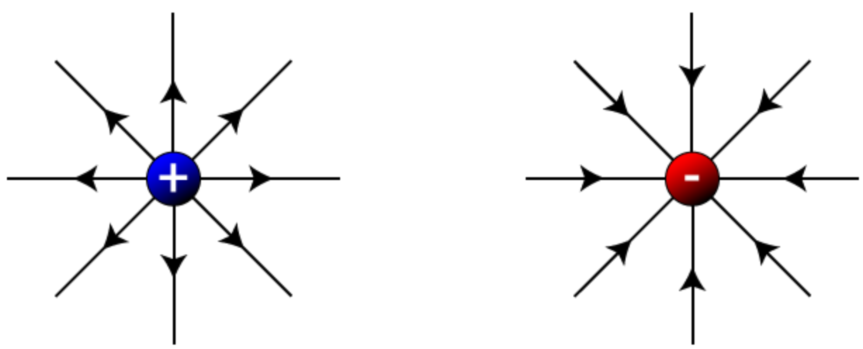


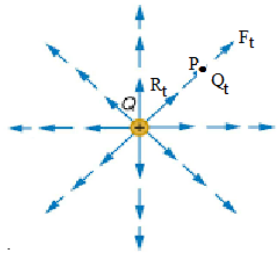
Fig 1.3 The lines of force due to a pair of positive charges



Electric field due to isolated positive charge & negative charge

An electric field is said to exist if a test charge kept in the medium which is at a distance $|\overrightarrow{R_t}|$, then it will experience a force F_t.

A point charge kept at the origin. Consider a point P which is at a distance $|\overrightarrow{R_t}|$ meters from the origin. A small test charge Q_t placed at the point P, then it experience a force F_t



Electric field intensity is defined mathematically as

$$\boldsymbol{E} = \lim_{Q_t \to 0} \frac{\boldsymbol{F_t}}{Q_t} \tag{1.10}$$

or simply

$$\boldsymbol{E} = \frac{\boldsymbol{F_t}}{Q_t} \tag{1.11}$$

The electric field intensity ***E*** is obviously in the direction of the force ***F*** and is measured in newton/coulomb or volts/meter. From the above figure the force experienced by the test charge Q_t is given by

$$\mathrm{F_t} = \frac{1}{4\pi\varepsilon} \frac{\mathrm{QQ_t}}{\overrightarrow{|\mathrm{R_t}|^2}} \overrightarrow{\mathrm{a_{R_t}}}$$

then electric field is

$$E = \frac{\boldsymbol{F_t}}{Q_t} = \frac{1}{4\pi\varepsilon} \frac{Q}{\overrightarrow{|R_t|^2}} \overrightarrow{a_{\boldsymbol{R_t}}}$$

$$\boldsymbol{E} = \frac{1}{4\pi\varepsilon} \frac{Q}{\overrightarrow{|R_t|^2}} \overrightarrow{a_{\boldsymbol{R_t}}} \tag{1.12}$$

Electric field due to N no. of charges:

If we have more than two point charges, we can use *the principle of superposition* to determine the force on a particular charge. The resultant force ***F*** on a charge Qlocated at point(p) is the vector sum of the forces exerted on Qby each of the charges Q_1, Q_2, Q_3.................... Q_n.Hence:

$$\boldsymbol{F} = \boldsymbol{F_1} + \boldsymbol{F_2} + \cdots + \boldsymbol{F_n}$$

From eq. (1.9)

$$\boldsymbol{F} = 9 \times 10^9 \, Q \sum_{i=1}^{N} \frac{Q_i}{R^3{}_{ip}} \boldsymbol{R_{ip}}$$

We know that

$$\boldsymbol{E} = \frac{\boldsymbol{F}}{Q}$$

$$\boldsymbol{E} = 9 \times 10^9 \sum_{i=1}^{N} \frac{Q_i}{R^3{}_{ip}} \boldsymbol{R}_{ip} \qquad (1.13)$$

ELECTRIC FIELDS DUE TO CONTINUOUS CHARGE DISTRIBUTIONS:

So far we have only considered forces and electric fields due to point charges, which are essentially charges occupying very small physical space. It is also possible to have continuous charge distribution along a line, on a surface, or in a volume as illustrated in figure 1.2.

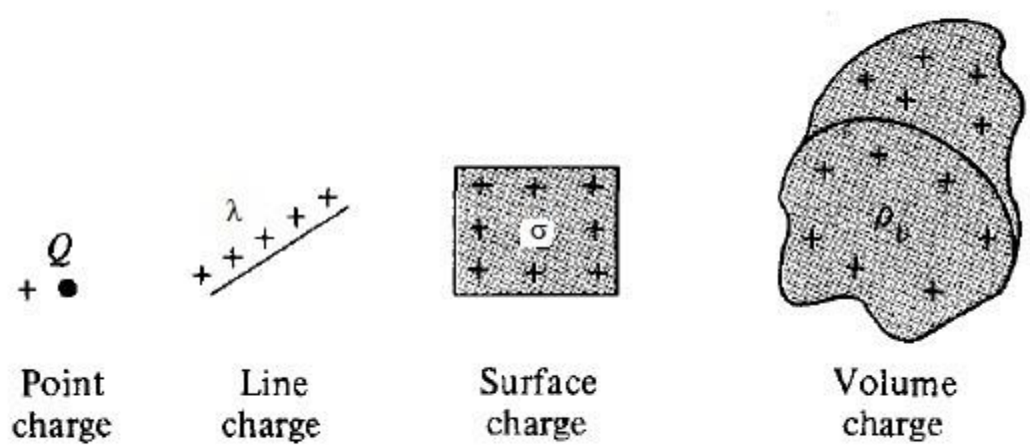


Fig. 1.4 volume charge distribution and charge elements

It is customary to denote the line charge density, surface charge density, and volume charge density by λ(in C/m), σ(in C/m^2), and ρ_v(in C/m^3), respectively.

Line charge density: when the charge is distributed over linear element, then the line charge density is the charge per unit length.

$$\lambda = \lim_{dl \to 0} \frac{dq}{dl}$$

Wheredq is the charge on a linear element dl.

Surface charge density: when the charge is distributed over surface, then the surface charge density is the charge per unit area.

$$\sigma = \lim_{ds \to 0} \frac{dq}{ds}$$

Wheredq is the charge on a surface element ds.

Volume charge density: when the charge is confined within a volume, then the volume charge density is the charge per unit volume.

$$\rho_v = \lim_{dv \to 0} \frac{dq}{dv}$$

Wheredq is the charge contained in a volume element dv.

Electric field due to line charge:

Consider a uniformly charged wire of length L m, the charge being assumed to be uniformly distributed at the rate of λ (linear charge density) c/m. Let P be any point at which electric field intensity has to be determined.

Consider a small elemental length dx at a distance x meters from the left end of the wire, the corresponding charge element is λ dx. Divide the wire into a large number of such small elements, each element will render its contribution towards the production of field at P.

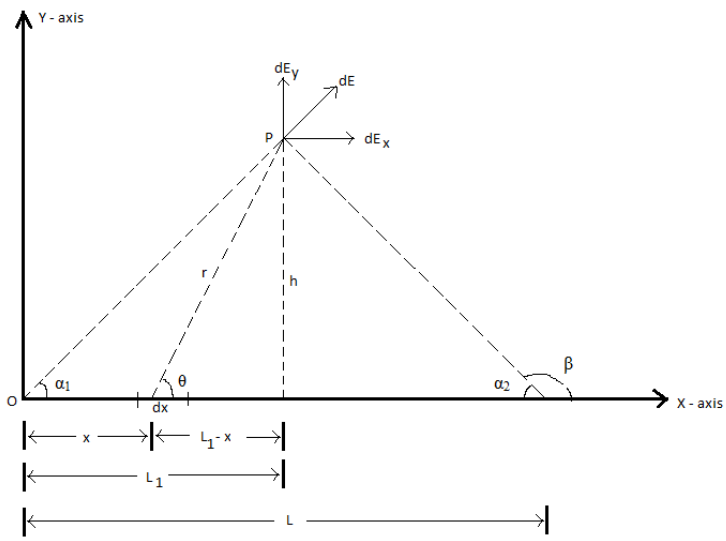


Fig 1.5 Evaluation **E** due to line charge

Let d**E** be field due to the charge element λ dx. It has a component dE_x along x-axis and dE_yalong y-axis.

d**E** = $dE_x\mathbf{a_x} + dE_y\mathbf{a_y}$

we know that

$$dE = \frac{\lambda\, dx}{4\pi\varepsilon_o r^2} \quad (1.14)$$

From figure 1.5 we can write,

$dE_x = dE\cos\theta$ (1.15)

$dE_y = dE\sin\theta$ (1.16)

Therefore we can write,

$$dE_x = \frac{\lambda \cos\theta \, dx}{4\pi\varepsilon_o r^2} \quad (1.17)$$

$$dE_y = \frac{\lambda \sin\theta \, dx}{4\pi\varepsilon_o r^2} \quad (1.18)$$

Substitutingeqs. (1.17) and (1.18) in d**E**we get,

$$d\boldsymbol{E} = \frac{\lambda \cos\theta \, dx}{4\pi\varepsilon_o r^2}\boldsymbol{a_x} + \frac{\lambda \sin\theta \, dx}{4\pi\varepsilon_o r^2}\boldsymbol{a_y} \quad (1.19)$$

From figure 1.5 we write

$$L_1 - x = h \cot\theta \quad (1.20)$$

$$-dx = -h \operatorname{cosec}^2\theta \, d\theta \quad (1.21)$$

$$r = h \operatorname{cosec}\theta \quad (1.22)$$

Substituting equations (1.20), (1.21) and (1.22) in (1.19), we get

$$d\boldsymbol{E} = \frac{\lambda \cos\theta \, d\theta}{4\pi\varepsilon_o h}\boldsymbol{a_x} + \frac{\lambda \sin\theta \, d\theta}{4\pi\varepsilon_o h}\boldsymbol{a_y} \quad (1.23)$$

The electric field intensity **E** due to whole length of the wire

$$\boldsymbol{E} = \int_{\boldsymbol{\theta=\alpha_1}}^{\boldsymbol{\theta=\pi-\alpha_2}} d\boldsymbol{E} \, d\theta$$

$$\boldsymbol{E} = \int_{\boldsymbol{\theta=\alpha_1}}^{\boldsymbol{\theta=\pi-\alpha_2}} \left[\frac{\lambda \cos\theta \, d\theta}{4\pi\varepsilon_o h}\boldsymbol{a_x} + \frac{\lambda \sin\theta \, d\theta}{4\pi\varepsilon_o h}\boldsymbol{a_y}\right] d\theta$$

$$\boldsymbol{E} = \frac{\lambda}{4\pi\varepsilon_o h}\left[\sin\theta \, \boldsymbol{a_x} - \cos\theta \, \boldsymbol{a_y}\right]_{\boldsymbol{\alpha_1}}^{\boldsymbol{\pi-\alpha_2}}$$

$$\boldsymbol{E} = \frac{\lambda}{4\pi\varepsilon_o h}\left[(\sin\alpha_2 - \sin\alpha_1)\boldsymbol{a_x} + (\cos\alpha_2 + \cos\alpha_1)\,\boldsymbol{a_y}\right]\frac{N}{C} \quad (1.24)$$

Case (i)

If P is the midpoint, $\alpha_1 = \alpha_2 = \alpha$

$$\boldsymbol{E} = \frac{\lambda}{4\pi\varepsilon_o h}\left[(\sin\alpha - \sin\alpha)\boldsymbol{a_x} + (\cos\alpha + \cos\alpha)\,\boldsymbol{a_y}\right]\frac{N}{C}$$

$$\boldsymbol{E} = \frac{\lambda}{4\pi\varepsilon_o h}\left[2\cos\alpha\,\boldsymbol{a_y}\right]\frac{N}{C}$$

$$\boldsymbol{E} = \frac{\lambda}{2\pi\varepsilon_o h}\cos\alpha\,\boldsymbol{a_y}\,\frac{N}{C} \quad (1.25)$$

The direction of electric field intensity is normal to the line charge. The electric field is not normal to the line charge if the point is not at midpoint.

Case (ii)

As length tendsto ∞

$\alpha_1 = 0$ and $\alpha_2 = 0$

From equation (1.24), we get

$$\boldsymbol{E} = \frac{\lambda}{2\pi\varepsilon_o h}\boldsymbol{a_y}\frac{N}{C} \qquad (1.26)$$

Electrical field due to charged ring:

A circular ring of radius '*a*'carries a uniform charge λC/m and is placed on the xy-plane with axis the same as the z-axis as shown in figure.

Let d**E** be the electric field intensity due to a charge dQ. The ring is assumed to be formed by several point charges. When these vectors are resolved, radial components get cancelled and normal components get added. Therefore the direction of electric field intensity is normal to the plane of the ring. The sum of normal components can be written as

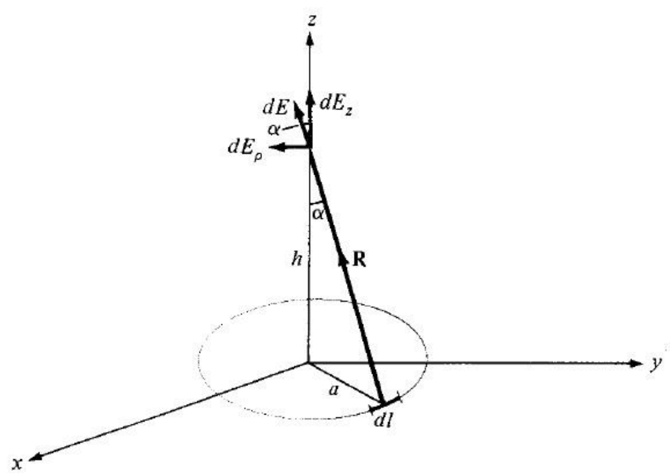


Fig. 1.6 charged ring

$$\boldsymbol{E} = \int dE cos\alpha\, \boldsymbol{a_z}$$

$$\boldsymbol{E} = \int \frac{dQ}{4\pi\varepsilon_o R^2} cos\ \alpha\, \boldsymbol{a_z}$$

$$\boldsymbol{E} = \int \frac{\lambda\, dl}{4\pi\varepsilon_o R^2}\frac{h}{R}\boldsymbol{a_z}$$

$$E = \frac{\lambda\, h}{4\pi\varepsilon_o R^3} \boldsymbol{a_z} \int dl$$

$$E = \frac{\lambda\, h}{4\pi\varepsilon_o R^3} \times 2\pi a\, \boldsymbol{a_z}$$

$$E = \frac{\lambda\, h}{4\pi\varepsilon_o \sqrt{a^2 + h^2}^3} \times 2\pi a\, \boldsymbol{a_z}$$

$$E = \frac{\lambda\, a\, h}{2\varepsilon_o (a^2 + h^2)^{3/2}} \boldsymbol{a_z} \frac{\mathrm{N}}{\mathrm{C}} \quad (1.27)$$

Electrical field due to a charged disc:

A disc of radius ‘a’ meters is uniformly charged with a charged density σ c/m^2. It is required to determine the electric field at ‘P’ which is at a distance h meters from the centre of the disc as shown in figure 1.7

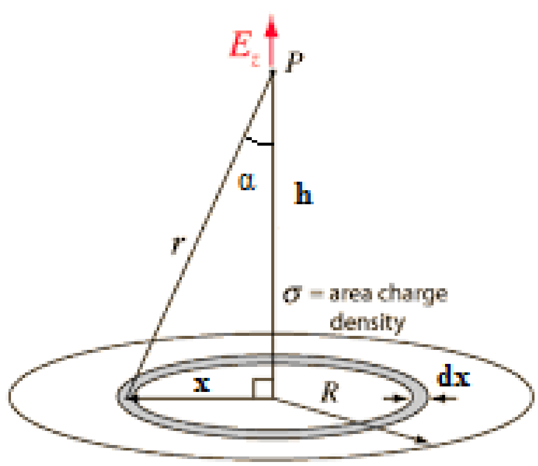


Fig. 1.7 charged disc

The disc is assumed to be formed by several rings of increasing radius. Consider a ring of radius x meters. Each ring is assumed to be formed by number of point charges.

Let $d\mathbf{E_1}$ be the electric field intensity due to a charge dQ_1 and $d\mathbf{E_2}$ is the electric field intensity due to a charge dQ_2.

Electric field due to one ring be obtained by adding normal components of $d\mathbf{E_1}$, $d\mathbf{E_2}$ .... $d\mathbf{E_n}$.

Therefore

$d\mathbf{E} = (dE_1\cos\theta + dE_2 \cos\theta + ... + dE_n\cos\theta\ )\ \mathbf{a_z}$

$d\mathbf{E} = (dE_1 + dE_2\ + ... + dE_n) \cos\theta \mathbf{a_z}$

d**E** = $\left(\frac{dQ_1}{4\pi\varepsilon r^2} + \frac{dQ_2}{4\pi\varepsilon r^2} + \ldots\ldots + \frac{dQ_n}{4\pi\varepsilon r^2}\right)\cos\theta\, \mathbf{a_z}$

d**E** = $\frac{dQ_1 + dQ_2 + \cdots + dQ_n}{4\pi\varepsilon r^2}\cos\theta\, \mathbf{a_z}$

d**E** = $\frac{dQ}{4\pi\varepsilon r^2}\cos\theta\, \mathbf{a_z}$

The total charge of the ring is σ ds which is equal to dQ

d**E**= $\frac{\sigma\, ds}{4\pi\varepsilon r^2}\cos\theta\, \mathbf{a_z}$(1.28)

ds = $\pi[(x + dx)^2 – x^2]$

ds = $\pi[x^2 + dx^2 + 2xdx – x^2] = 2\pi\, x\, dx$ (neglecting dx^2 term)

Substituting ds in eq. (1.28) , we get

d**E** = $\frac{\sigma\, 2\pi x dx}{4\pi\varepsilon r^2}\cos\theta\, \mathbf{a_z}$

d**E** = $\frac{\sigma\, xdx}{2\varepsilon r^2}\cos\theta\, \mathbf{a_z}$(1.29)

From above figure we can write

Tanθ = x/h

x = h tanθ (1.30)

dx = h $\sec^2\theta\, d\theta$ (1.31)

cosθ = h/r

r = h/ cosθ = h secθ (1.32)

Substituting equations (1.30), (1.31) and (1.32) in (1.29) we get,

$$d\mathbf{E} = \frac{\sigma\,(h \tan\theta)(h \sec^2\theta\, d\theta)}{2\varepsilon h\, \sec\theta^2}\cos\theta\, \mathbf{a_z}$$

$$d\mathbf{E} = \frac{\sigma}{2\varepsilon}\sin\theta\, d\theta\, \mathbf{a_z}$$

On integrating

$$\boldsymbol{E} = \frac{\sigma}{2\varepsilon}\int_0^{\alpha} \sin\theta\ d\theta\ \boldsymbol{a_z}$$

$$\boldsymbol{E} = \frac{\sigma}{2\varepsilon}[-\cos\theta]_0^{\alpha}\boldsymbol{a_z}$$

$$\boldsymbol{E} = \frac{\sigma}{2\varepsilon}(1 - \cos\alpha)\boldsymbol{a_z} \qquad (1.33)$$

From figure 1.7

$$cos\alpha = \frac{h}{\sqrt{(R^2 + h^2)}}$$

$$\therefore \boldsymbol{E} = \frac{\sigma}{2\varepsilon}\left(1 - \frac{h}{\sqrt{(R^2 + h^2)}}\right)\boldsymbol{a_z}\frac{N}{C} \tag{1.34}$$

For infinite disc, radius ‘R’ tends to infinite and α = 90.

$$\boldsymbol{E} = \frac{\sigma}{2\varepsilon}(1 - cos\ 90)\boldsymbol{a_z}$$

$$\boldsymbol{E} = \frac{\sigma}{2\varepsilon}\boldsymbol{a_z}\frac{N}{C}or\frac{V}{m} \tag{1.35}$$

From equation (1.35), it can be seen that electric field due to infinite disc is independent of distance. Electric field is uniform.

ELECTROSTATICS-II

Learning objectives:

- To introduce Gauss's law with its applications
- To familiarize students work done in moving a point charge, Electric Potential and Potential gradient

Syllabus:

Gauss's law, Max well's First equation, Div (D) = ρ_V. Application of Gauss's Law, Problems on Gauss's law, Work done in moving a point charge in an Electrostatic field. Electric potential, Properties of a potential function, Electric potential gradient

Learning outcomes:

Students will be able to

- Apply Gauss law for finding EFI and Electric flux density
- Determine the work done by a point charge placed in electric field..
- Define electric potential and potential difference and derive expressions potential and potential difference for line charge, circular ring and charged disc.
- Evaluate the electric field from potential (Potential gradient)

ELECTROSTATICS-II

Electric flux (or) displacement flux:

The total number of lines of force in any particular electric field is called electric flux. It is denoted by symbol Ψ. Similar to the charge the unit of electric flux is also coulomb.

Properties of Electric flux lines:

The electric flux is nothing but the lines of force, around a charge. Such electric flux lines have the following properties

1. Electric flux lines start from positive charge and terminate on the negative charge as shown in fig 2.1

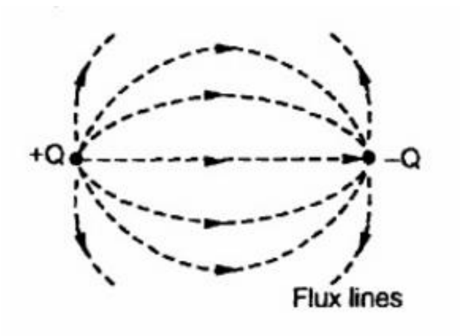


Fig 2.1 Flux lines

2. If the negative charge is absent, then the total flux lines terminate at ∞ as shown in fig 2.2(a). While in absence of positive charge, the electric flux lines terminates on the negative charge from ∞ as shown in fig 2.2(b)

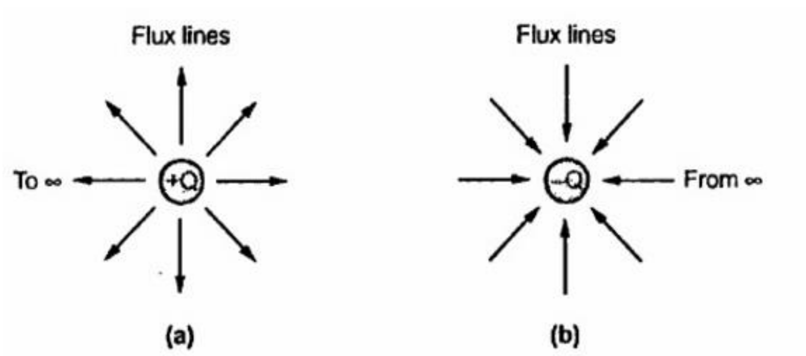


Fig 2.2

3. If there are more number of flux lines i.e crowding of flux lines the E is stronger.
4. Electric flux lines are parallel and never cross each other.
5. The electric flux lines are independent of medium in which charges are placed.
6. The electric flux lines always enter or leave the charged surface normally.

Electric flux density (or) displacement flux density:

The net flux passing normal through the surface per unit area is called electric flux density. It is denoted as $\overline{D}$. It has a specific direction which is normal to the surface area under consideration hence it is a vector field.

$$\overline{D} = \frac{\Psi}{A}\overrightarrow{a_r}$$

Where Ψ = Flux passing through the surface.

A = Surface area.

$\overrightarrow{a_r}$= Unit vector normal to the plane of surface.

The units of D are C/m^2.

D due to a point charge Q:

Consider a point charge +Q placed at the centre of the imaginary sphere of radius r as shown in fig 2.3.

The flux lines originating from the point charge +Q are directed radially outwards. The magnitude of flux density at any point on the surface is,

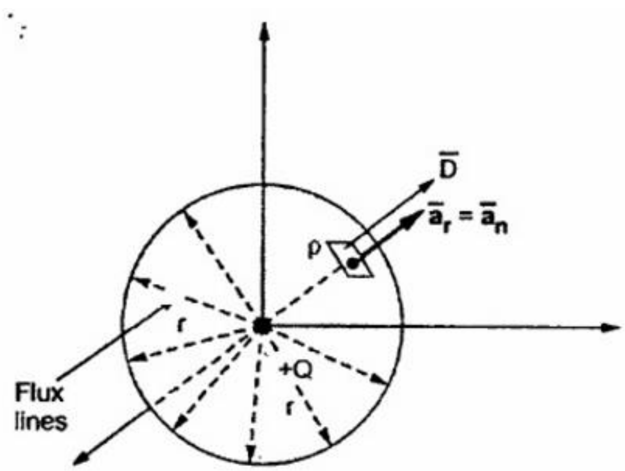


Fig 2.3

$$\overline{|D|} = \frac{Total flux \Psi}{Total surface area A}$$

But Ψ = Q and A= $4\pi r^2$

$$\overline{D} = \frac{Q}{4\pi r^2}\overrightarrow{a_r} C/m^2.$$

Relation between $\bar{D}$ and $\bar{E}$:

Consider a charge is kept at the origin or centre of spherical cell, the electric field on the surface of spherical cell is given by

$$\bar{E} = \frac{Q}{4\pi\varepsilon r^2}\overrightarrow{a_r}$$

The electric flux density on the surface of spherical cell is given by

$$\bar{D} = \frac{Q}{4\pi r^2}\overrightarrow{a_r}$$

Dividing equations of $\bar{D}$ and $\bar{E}$ we get

$$\frac{\bar{D}}{\bar{E}} = \varepsilon$$

$$\bar{D} = \varepsilon\bar{E}$$

Gauss's Law:

Statement : "The total normal electric flux over a closed surface in an electric field is equal to the total charge enclosed by that surface ".

(or)

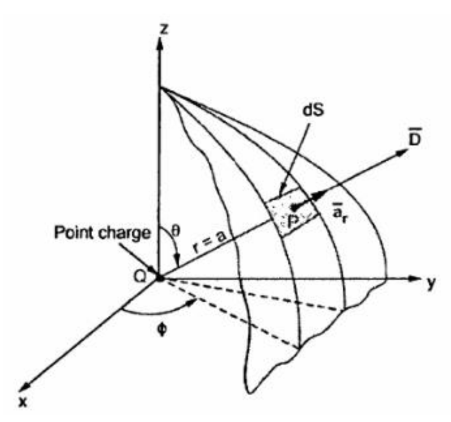
Flux coming out of a charged body is equal to the amount of charge contained by that body.

Mathematically it may be expressed as

$$\int D.ds = Q$$

Where Q is the total charge enclosed. The surface over which the integral is taken is called Gaussian surface.

Proof: Consider a point Q kept at the origin as shown in figure. Consider point P at a distance r m from the origin. The displacement density at this point is D. The direction of vector ds is normal to the area.



Let the flux through the area dS be dΨ.

Displacement flux density $D = \frac{d\Psi}{dS}$

$$d\Psi = D\ dS$$

Total flux coming out of the spherical surface can be obtained by integration

$$\int d\Psi = \int D.dS$$

$$\Psi = \iint D.dS$$

From faraday's experiment we know that

$$D = \frac{Q}{4\pi r^2}$$

$$\Psi = \iint \frac{Q}{4\pi r^2} dS$$

Total surface area $4\pi r^2$

$$\Psi = \frac{Q}{4\pi r^2} X 4\pi r^2 = Q$$

$$\text{Therefore } \Psi = Q$$

Applications of Gauss's law:

It is used to find the value of electric field intensity E and Electric flux density D. Construct an imaginary surface such that electric field is uniform and it is normal to the surface at every point such surface is called Gaussian surface. Apply Gauss's law to the Gaussian surface.

Electric field due to point charge:

Construct a Gaussian surface of radius r and apply Gauss's law

$$\Psi = Q$$

$$\int D.dS = Q$$

$$D\int dS = Q$$

$$D.4\pi r^2 = Q$$

$$D = \frac{Q}{4\pi r^2}$$

$$E = \frac{Q}{4\pi \varepsilon r^2}$$

Electric field intensity due to infinite line charge:

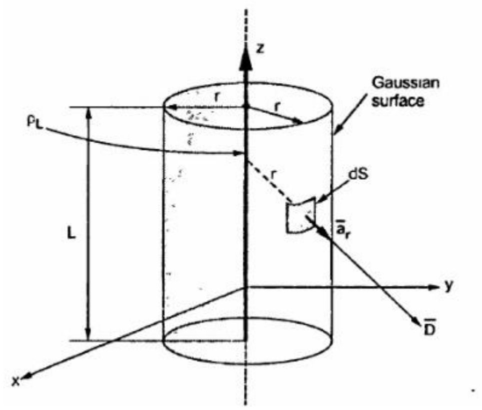


Fig : Infinite Line Charge

Consider a cylindrical surface of radius r m and height L m. The line charge can be assumed to be formed by several point charges. Therefore direction of resultant electric field vector is normal to the line charge.

Applying Gauss's law

$$\Psi = Q$$

$$\Psi_1+\Psi_2+\Psi_3 = Q$$

Flux coming out of surfaces 2 and 3 is zero

Since flux is purely normal ($\int D.ds = 0$). Since angle between them is 90^0

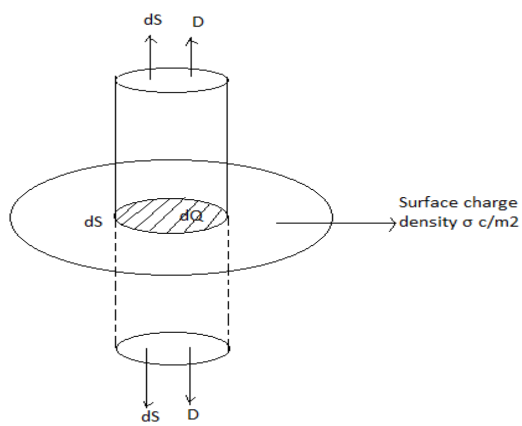
$$D\ 2\pi rl=\lambda l$$

$$D= \lambda/2\pi r$$

$$\frac{\overline{D}}{\overline{E}} = \varepsilon$$

$$E = \frac{D}{\varepsilon} = \frac{\lambda}{2\pi\varepsilon r}$$

Electric field due to infinite charged disc:



Consider an infinite disc of surface charge density σ c/m^2. The E for this infinite disc is always normal to the plane of surface.

Construct a pill box as shown in fig and apply Gauss's law to pillbox.

$$\Psi = Q$$

$$\Psi_{top} + \Psi_{bottom} + \Psi_{sides} = \sigma dS$$

Flux from the sides is zero because the E and surface is perpendicular to each other.

$$\Psi_{top} + \Psi_{bottom} = \sigma dS$$

$$Dds + Dds = \sigma dS$$

$$D = \sigma/2$$

$$E = D/\varepsilon$$

$$E = \sigma /2\varepsilon \, a_r$$

E due to infinite disc is independent of distance.

Gauss's law cannot be applied to finite disc because E is not uniform.

Poisson's and Laplace's Equations:

From the Gauss law we know that

$$\int D.ds = Q \quad (1)$$

A body containing a charge density **ρ** uniformly distributed over the body. Then charge of that body is given by

$$Q = \int \boldsymbol{\rho} dv \quad (2)$$

$$\int D.ds = \int \boldsymbol{\rho} dv \quad (3)$$

This is integral form of Gauss law.

As per the divergence theorem

$$\int D.ds = \int \boldsymbol{\nabla} . \mathbf{D} dv \quad (4)$$

$$\boldsymbol{\nabla} . \mathbf{D} = \boldsymbol{\rho} \quad \mathbf{(5)}$$

This is known as point form or vector form or polar form. This is also known as Maxwell's first equation.

$$D = \varepsilon E \quad (6)$$

$$\nabla . \mathcal{E}E = \rho$$

$$\nabla . E = \rho/\mathcal{E} \quad (7)$$

We know that E is negative potential medium

$$E = -\nabla V \quad (8)$$

From equations 7 and 8

$$\nabla . (-\nabla V) = \rho/\mathcal{E}$$

$$\nabla^2 V = -\rho/\mathcal{E} \quad (9)$$

Which is known as Poisson's equation in static electric field.

Consider a charge free region (insulator) the value of **ρ** = 0, since there is no free charges in dielectrics or insulators.

$$\nabla^2 V = 0$$

This is known as Laplace's equation.

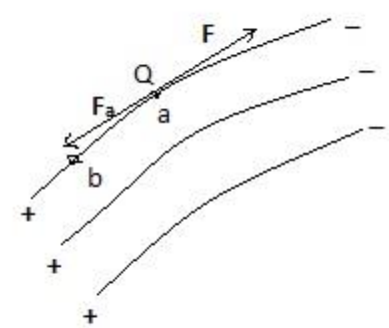
Cartesian coordinate system

$$\nabla^2 V = \frac{\partial 2}{\partial x} V$$

Work Done:

If a point charge 'Q' is kept in an electric field it experiences a force **F** in the direction of electric field. $\mathbf{F_a}$ is the applied force in a direction opposite to that of **F**.

Let dw be the work done in moving this charge Q by a distance *dl* m. Total work done in moving the point charge from 'a' to 'b' can be obtained by integration.



$$W = \int dw$$

$$W = \int \boldsymbol{F_a} \cdot \boldsymbol{dl}$$

$$W = -\int_a^b \boldsymbol{F} \cdot \boldsymbol{dl} \qquad (1.36)$$

$$\mathbf{E} = \mathbf{F}/Q$$

$$\mathbf{F} = \mathbf{E}\,Q$$

$$W = -\int_a^b \boldsymbol{E}\,Q \cdot \boldsymbol{dl}$$

$$W = -Q\int_a^b \boldsymbol{E} \cdot \boldsymbol{dl} \qquad (1.37)$$

$$\mathbf{E} = E_x\mathbf{a_x} + E_y\mathbf{a_y} + E_z\mathbf{a_z}$$

$$d\boldsymbol{l} = dx\,\mathbf{a_x} + dy\mathbf{a_y} + dz\mathbf{a_z}$$

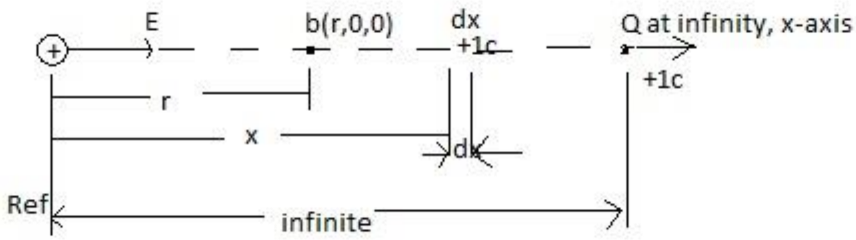
$$\mathbf{E} \cdot d\boldsymbol{l} = E_X\,d_x + E_y d_y + E_z d_z$$

$$\therefore W = -Q\int_{(x_1,y_1,z_1)}^{(x_2,y_2,z_2)} (E_x dx + E_y dy + E_z dz) \qquad (1.38)$$

Absolute Potential:

Absolute potential is defined as the work done in moving a unit positive charge from infinite to the point against the electric field.

A point charge Q is kept at an origin as shown in figure. It is required to find the potential at ‘b’ which is at distance ‘r’ m from the reference.



Consider a point R_1 at a distance 'x' m. The small work done to move the charge from R_1 to P_1 is dw. The electrical field due to a point charge Q at a distance 'x' m is

$$\boldsymbol{E} = Q/(4\pi\varepsilon x^2)$$

Work done = **E**• d**x**

Total work done can be obtained by integration

Work done (W) = $-\,\mathrm{Q}\int_a^b \mathbf{E}\cdot d\mathbf{l}$

$$\mathrm{V} = -1\int_a^b \mathbf{E}\cdot d\mathbf{l}$$

$$\mathrm{V} = -\int_a^b \mathbf{E}\cdot d\mathbf{l}$$

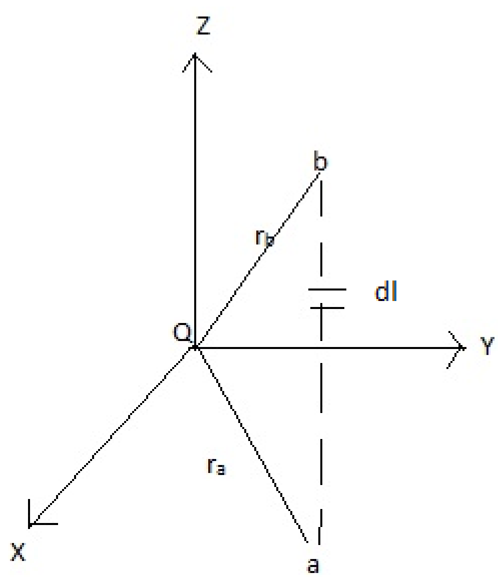
$\mathbf{E} = E_x\mathbf{a_x}$, $d\mathbf{l} = dx\,\mathbf{a_x}$

$$\mathrm{V} = -\int_a^b \frac{Q}{4\pi\varepsilon x^2}\,dx$$

$$V = \frac{Q}{4\pi\varepsilon r} \qquad (1.39)$$

Potential Difference: V_{ab}

Potential difference V_{ab} is defined as the work done in moving a unit positive charge from 'b' to 'a'.



Consider a point charge Q kept at the origin of a spherical co-ordinate system. The field is always in the direction of $\mathbf{a_r}$. No field in the direction of θ and ϕ. The points ‘a’ and ’b’ are at distance r_a and r_b respectively as shown in figure.

$$V_{ab} = -\int_{b}^{a} \mathbf{E} \cdot d\mathbf{l}$$

$\mathbf{E} = E_r\mathbf{a_r}$,$\mathbf{dl} = dr\mathbf{a_r}$ and $E.dl= E_r d_r$

$$V_{ab} = -\int_{b}^{a} E_r d_r$$

$$V_{ab} = -\int_{r_b}^{r_a} \frac{Q}{4\pi\varepsilon r^2}\, d_r$$

$$V_{ab} = \frac{Q}{4\pi\varepsilon}\left[\frac{1}{r_a} - \frac{1}{r_b}\right]$$

$$V_{ab} = \frac{Q}{4\pi\varepsilon}\frac{1}{r_a} - \frac{Q}{4\pi\varepsilon}\frac{1}{r_b}$$

$$V_{ab} = V_a - V_b \qquad (1.40)$$

Potential difference due to line charge:

The wire is uniformly charged with λ C/m. We have to find the potential difference V_{ab} due to this line charge. Consider a point P at a distance P from the line charge.

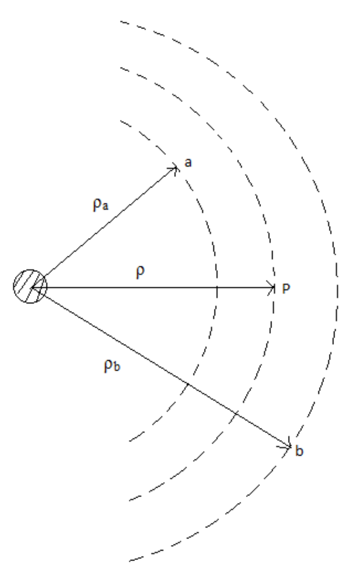


Fig. Line charge

$\mathbf{E} = E_\rho \mathbf{a}_\rho$

$d\mathbf{l} = d\rho \mathbf{a}_\rho$

$\mathbf{E} \bullet d\mathbf{l} = E_\rho \mathbf{a}_\rho \bullet d\rho \mathbf{a}_\rho = E_\rho d\rho$

Potential difference V_{ab} is the work done in moving a unit +ve charge from 'b' to 'a'.

$$V_{ab} = -\int_b^a \mathbf{E} \cdot d\mathbf{l}$$

$$V_{ab} = -\int_b^a E\rho \, d\rho$$

$$V_{ab} = -\int_{\rho_b}^{\rho_a} \frac{\lambda}{2\pi\varepsilon\rho} \, d\rho$$

$$V_{ab} = \frac{\lambda}{2\pi\varepsilon} \ln \frac{\rho_b}{\rho_a} \qquad (1.41)$$

Potential due to charged ring:

A thin wire is bent in the form of a circular ring as shown in figure. It is uniformly charged with a charge density λ C/m. It is required to determine the potential at height 'h' meters from the centre of the ring. The ring is assumed to be formed by several point charges.

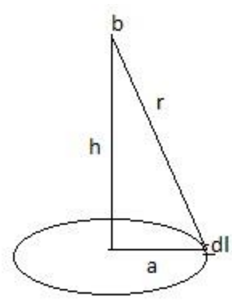


Fig. Charged ring

Let dv be the potential due to the charge element of length dl containing a charge dQ.

$$dv = \frac{dQ}{4\pi\varepsilon r}$$

$$dv = \frac{\lambda dl}{4\pi\varepsilon r}$$

$$V = \int \frac{\lambda dl}{4\pi\varepsilon r}$$

$$V = \frac{\lambda}{4\pi\varepsilon r}\int dl$$

$$V = \frac{\lambda}{4\pi\varepsilon r}\ 2\pi a$$

$$V = \frac{\lambda a}{2\varepsilon r}$$

$$V = \frac{\lambda a}{2\varepsilon\sqrt{a^2 + b^2}}\ \ volts \qquad (1.42)$$

Potential due to a charged disc:

let dv be the potential due to one ring. Each ring is assumed to be having several point charges dQ_1, dQ_2, ...dQ_n. Potential due to the entire ring is the sum of potential values due to each point charge.

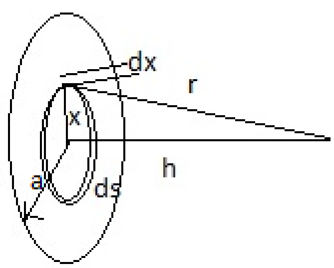


Fig. Charge disc

$$dv = \frac{dQ_1}{4\pi\varepsilon r} + \frac{dQ_2}{4\pi\varepsilon r} + \ldots\ldots\ldots\ldots\ldots. + \frac{dQ_n}{4\pi\varepsilon r}$$

$$dv = \frac{dQ_1 + dQ_2 + \ldots\ldots.. + dQ_n}{4\pi\varepsilon r}$$

$$dv = \frac{dQ}{4\pi\varepsilon r}$$

$$dv = \frac{\sigma\, ds}{4\pi\varepsilon r} = \frac{\sigma\, 2\pi x\, dx}{4\pi\varepsilon r} = \frac{\sigma\, x\, dx}{2\varepsilon r}$$

Potential due to entire disc can be obtained by integration

$$V = \int dv = \int_0^a \frac{\sigma\, x\, dx}{2\varepsilon r} = \frac{\sigma}{2\varepsilon}\int_0^a \frac{x}{\sqrt{x^2 + h^2}}\, dx$$

Let $x^2 + h^2 = t^2$

2x dx = 2t dt

Therefore we have

$$V = \frac{\sigma}{2\varepsilon}\int_{h}^{\sqrt{a^2+h^2}} \frac{t\,dt}{t}$$

$$V = \frac{\sigma}{2\varepsilon}\int_{h}^{\sqrt{a^2+h^2}} dt$$

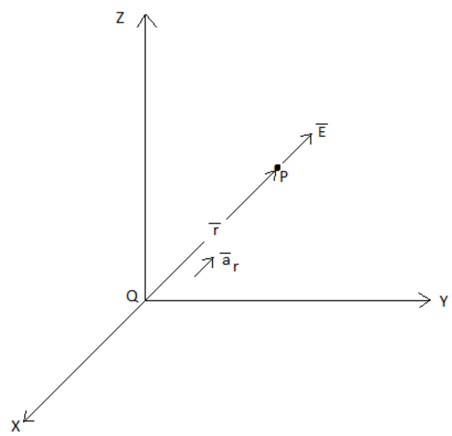
$$V = \frac{\sigma}{2\varepsilon}\left(\sqrt{a^2+h^2} - h\right)\ volts \tag{1.43}$$

At the centre of the disc , h=0;

$$V = \frac{\sigma a}{2\varepsilon}\ \ volts \tag{1.44}$$

Relation between V and E:

Consider a point charge Q at the origin as shown in figure. Electric field due to this charge at the point ‘P’ is



$$\boldsymbol{E} = \frac{Q}{4\pi\varepsilon r^2}\boldsymbol{a_r}$$

Consider

$$\boldsymbol{\nabla}\left(\frac{1}{r}\right) = \left(\frac{\partial}{\partial r}\boldsymbol{a_r} + \ \ldots\ldots\ldots\right)\left(\frac{1}{r}\right)$$

$$\boldsymbol{\nabla}\left(\frac{1}{r}\right) = -\left(\frac{1}{r^2}\right)\boldsymbol{a_r}$$

Using above expression in **E** , we get

$$\boldsymbol{E} = \frac{Q}{4\pi\varepsilon} - \boldsymbol{\nabla}\left(\frac{1}{r}\right)$$

$$\boldsymbol{E} = -\boldsymbol{\nabla}\frac{Q}{4\pi\varepsilon}\left(\frac{1}{r}\right)$$

Therefore,

$$\boldsymbol{E} = -\boldsymbol{\nabla}\, V$$

UNIT - III

Electric Dipole:

It is defined as two equal and opposite charges separated by a small distance.

Electric Dipole moment:

It is defined as product of charge Q and distance between the two charges. It is a vector since length is in vector, length vector is directed from negative to positive charge, therefore dipole moment is from negative to positive charge.

$$m = Ql$$

$$l = l\, u_r$$

u_r is unit vector directed from negative to positive charge. The unit of dipole moment is Cm

Potential due to electric dipole:

we have to find the potential at P which is at a distance r from the centre of the dipole. The distance from +Q and –Q to the point P are r_1 and r_2

$$r_1 = r - \frac{l}{2}\cos\theta$$

$$r_2 = r + \frac{l}{2}\cos\theta$$

Let V_1 be the potential due to +q, V_2 be the potential due to –Q.

$$V_1 = \frac{Q}{4\pi\varepsilon r1}$$

$$V_2 = \frac{-Q}{4\pi\varepsilon r2}$$

By the superposition theorem, total potential due to dipole

$$V = V_1+V_2$$

$$V = \frac{Q}{4\pi\epsilon}\frac{r2-r1}{r1r2}$$

$$r_2 - r_1 = l\cos\theta$$

$$r_2r_1 = r^2$$

$$V = \frac{\mathrm{Ql}\cos\theta}{4\pi r2}$$

M is magnetic dipole moment

We know that

$$E = -\nabla \mathbf{V}$$

Torque experienced by Dipole in uniform Electric Field:

There are two charges +Q and –Q forming a dipole, placed in uniform E. Each charge will experience a force equal in magnitude QE but oppositely directed and resultant force experienced by dipole zero because as F1 and F2 neutralize each other but these forces form a couple whose torque is equal to magnitude oppose into perpendicular distance between the couple charges.

$$T = dxF$$

$$\sin\theta = d/l$$

$$d = l \sin\theta$$

$$T = l \sin\theta \cdot F$$

$$T = Ql \sin\theta \cdot E$$

$$T = mxE$$

The torque is maximum when E and dipole moment are perpendicular to each other. The torque is minimum when E and dipole moment are parallel. So we conclude that dipole in uniform E does not experience translational forces. It experiences a force tending aligned the dipole axes with the E.

Current and current density:

Current through a given area is the electric charge passing through the area per unit time.

$$I = -\frac{dQ}{dt}$$

'-' indicates the opposite direction of electrons to the current.

Conduction current or Drift current:

The current flow due to the flow of free electrons in the conductor under the influence of applied voltage is called drift current. It obeys Ohms law.

Convection Current or displacement current:

The current due to the flow of charge under the influence of electric field is called convection current. It does not obey Ohms law.

Diffusion Current:

The current due to the movement of free electrons and free holes in semi conductor is called diffusion current.

Current Density:

The amount of current passing through a conductor is normal to the area of cross section / unit area is called current density. It is given by J.

When a steady current is passing through conductor, the current density is uniform and conduction has uniform cross section but J is different at different points if the conduction is non uniform cross section. The current density is represented by vector J. The unit is A/m^2

$$J = \lim_{s \to \Delta s} \frac{\Delta J}{\Delta S}$$

$$= I/S$$

$$= dI/dS$$

$$dI = J\,dS$$

$$I = \int J.dS$$

Depending upon the current is produced there are 2 types of J

1. Conduction current density or point form of Ohms law
2. Convection current density

Conduction current density or Point form of Ohms law:

Conduction current requires conductors.

Ohms law:
The current flowing through a linear circuit is directly proportional to impressed voltage provided the temperature is kept constant.

$$I\ \alpha\ V$$

$$I = GV$$

$$I = V/R$$

We know R= $\rho l/A$

$$I = \frac{VA}{\rho l}$$

$$\frac{I}{A} = \frac{1}{\rho}\frac{V}{l}$$

$$J = \sigma.\ E$$

Which is known as point form of Ohms law for insulators or dielectrics.
J = 0 conduction current cannot flow through free space.

Convection current density:

Convection current does not involve conductor and it does not obey Ohms law. It occurs and current flowing through a insulating medium or through liquid or through vaccum. Consider a current filament as shown. There is a flow of charge density ρ_v at velocity v along y-axis.

$$v = v_y \, a_y$$

The current through the filament

$$\Delta I = \frac{\Delta Q}{\Delta t}$$

We know $\rho_v = \frac{\Delta Q}{\Delta v}$

$$\Delta Q = \rho_v \Delta V$$

$$\Delta I = \rho_v \frac{\Delta V}{\Delta t} = \rho_v \frac{\Delta S \Delta l}{\Delta t}$$

$$\frac{\Delta I}{\Delta S} = \rho_v \frac{\Delta l}{\Delta t}$$

$$J = \rho_v v_y a_y$$

Here ΔI is the convection current and J is the convection current density.

Continuity Equation:

Continuity equation of charge works on the principle of law of conservation of charge. It states that the charge can neither be created nor destroyed. We know that current is rate of flow of charge.

$$I = -\frac{dQ}{dt} \qquad (1)$$

'-' indicates the opposite direction of electrons to the current.

We know that volume charge density

$$\rho_v = \frac{dQ}{dV}$$

$$Q = \int \rho_v dV \qquad (2)$$

From equations 1 & 2

$$I = -\int \frac{d\rho_v}{dt} dV \qquad (3)$$

We know

$$I = \int J . dS \qquad (4)$$

From 3 & 4

$$\int J . dS = - \int \frac{d\rho_v}{dt} dV$$

$$\int \nabla . J \, dV = - \int \frac{d\rho_v}{dt} dV$$

$$\nabla . J = - \frac{d\rho_v}{dt}$$

Boundary conditions between conductors and free space(Dielectric):

First Boundary condition:

Bigger source a boundary formed by conductor and free space, the charge cannot reside inside a conductor since they repel each other and finally they reach the boundary of the conductor. Construct a rectangular path ABCDA as shown. We know that electric field is conservative field

$$\int E . dl = 0$$

$$\int_{AB} E . dl + \int_{BC} E . dl + \int_{CD} E . dl + \int_{DA} E . dl = 0$$

$$E_{t1} \Delta l - E_{h1} \frac{\Delta h}{2} + 0 + 0 + 0 + E_{n1} \frac{\Delta h}{2} = 0$$

$$E_{t1} = 0$$

$$E = E_{t1} + E_{n1}$$

$$= E_{n1}$$

Electric field is always normal to the surface of the conductor.

Second boundary condition:

Construct a pill box and apply Gauss law to the pill box.

$$\varphi = Q$$

$$\int D . ds = Q$$

$$\int_{top} D . ds + \int_{bottem} D . ds + \int_{lateral} D . ds = \sigma dS$$

$$D_{n1} = \sigma$$

Normal component of flux density is equal to the normal flux density.

Properties of Conductors:

Electric field inside the conductor is zero.

Electric field is always normal to the surface of the conductor.

The value of electric flux density is equal to surface charge density.

The tangential component of electric field is zero.

Conductors & Dielectrics:

Conductor is one in which the outer electrons of an atom is easily detachable and migrate with application of weak Electric field.

A dielectric is one in which the electrons are rigidly bounded to their nucleus, so the ordinary electric field will not be able to detach them away. The dielectric placed in electrostatic field will be subjected to electro static induction. The electric field will twisted and strain the molecules to orient the positive charges in the direction of electric field and negative charges oppositely. If the electric field strength is too high the dielectric will break down cease to beam insulator.

Types of Dielectrics:

1. Polar dielectrics
2. Non-polar dielectrics

Polar dielectrics:

In polar dielectrics the molecules from dipoles even in absence of electric field. Even in absence of electric field, the dipoles are disposed at random the resultant electric field is zero. On the application of electric field the dipoles rearranged themselves so that their axes are aligned with the applied field. The electric field will twist and strain the molecules to orient the positive charges in the direction of electric field and negative charges oppositely. This shifting results an instantaneous current called displacement current which causes in very small fraction of seconds.

Eg: water, ether, ammonia

Non-Polar dielectrics:

In these dielectrics the positive and negative elements in the uncharged conditions are closed to each other that their action is neutral. In the application of electric field will strict the positive and negative charges lightly with in the molecules to give rise to dipole.

Eg: H, O etc

Polarization:

The elastic shifting of charged clouds in an atom of dielectric material when it is subjected to an electric field is called polarization. It is defined as movement of dipole.

$P = \frac{m}{V}$

If there are n dipoles the volume then total dipole moment is

$m = m_1+m_2+\ldots+m_n\Delta V$

$m = \sum_{i=1}^{n\Delta V} m_i$

polarization = total displacement / volume

$= \frac{\sum_{i=1}^{n\Delta V} m_i}{V}$

Dielectric Parameters:

Consider a dielectric material cutting the form of a slab of permittivity ϵas shown and placed in uniform electric field. The effect of field due to polarize the dielectric inducing atomic dipole through out the volume of specimen in the alignment with the electric field. Consequently neutralization of equal and opposite charge inside the dielectric charges reside on the slab and form dipole.

Polarization $P = \frac{Ql}{V} = \frac{Ql}{Al} = \frac{Q}{A} = \sigma_p\ u_l$

$\sigma_p$is surface charge density.

The internal field $E_i = E_a+E^1$

Where E_a is applied field, E^1 is field induced in the slab which is opposite to that of applied field.

$E^1 = -\frac{\sigma_p}{\epsilon_0} u_l$

$= -\frac{P}{\epsilon_0} u_l$

$E_i = E_a - \frac{P}{\epsilon_0}$

$E_a = E_i + \frac{P}{\epsilon_0}$

$\epsilon_0 E_a = \epsilon_0 E_i + P$

$D = \epsilon_0 E_i + P$ (1)

P α E_i

P = $\epsilon_0 \psi_p E_i$

D = ϵ_0 E_i + $\epsilon_0 \psi_p E_i$

D = $\epsilon_0 E_i(1+ \psi_p)$

D = $\epsilon_0 \epsilon_r$ E_i

Susceptibility (ψ_e) :

Number of dipoles induced by unit volume under the influence of unit strength electric field in a material is known as electrical susceptibility.

ψ_e = ϵ_r - 1

Susceptibility is one less than relative permittivity. For linear dielectric,

Polarization α E_i

P = $\epsilon_0 \psi_e E_i$ (1)

We know that D = $\epsilon_0 \epsilon_r$ E_i

D = $\epsilon_0(1+ \psi_e)$ E_i (2)

From equations 1 & 2

$\frac{P}{D} = \frac{\psi_e}{(1+\psi_e)} = \frac{\psi_e}{\epsilon_r}$ (3)

Dielectric Boundary conditions:

First boundary condition:

When the flux lines are flow through single medium they are continuous. If they go through boundary formed by two dielectrics they get reflected. First boundary condition deals with electric field intensity.

E_1 and E_2 are electric field in medium 1 and 2 respectively. Construct a rectangular path ABCDA as shown. And apply conservative property for the rectangular loop ABCDA.

$$\int E.dl = 0$$

$\int_{AB} E.dl + \int_{BC} E.dl + \int_{CD} E.dl + \int_{DA} E.dl = 0$

$E_{t1}\Delta l - E_{n1}\frac{\Delta h}{2} - E_{n2}\frac{\Delta h}{2} - E_{t2}\Delta l + E_{n2}\frac{\Delta h}{2} + E_{n1}\frac{\Delta h}{2} = 0$

$E_{t1} = E_{t2}$ (1)

At the boundary the tangent along components of electric field vectors are equal.

$\sin\theta_1 = \frac{Et1}{E_1}$

$E_{t1} = E_1 \sin\theta_1$ (2)

$E_{t2} = E_2 \sin\theta_2$ (3)

$E_1 \sin\theta_1 = E_2 \sin\theta_2$ (4)

Second boundary equations:

D_{n1} and D_{n2} are normal components of flux density vectors in medium 1 and 2 respectively. An infinite sheet with charge density σ C/m^2 is at the boundary. Second boundary condition deals with flux density. Construct the pill box at the boundary as shown. Apply Gauss's law

Flux enter the pill box $= D_{n2}dS$

Flux leave the pill box $= D_{n1}dS$

Net flux in the pill box $= D_{n2}dS - D_{n1}dS = \sigma$ dS

$D_{n2} - D_{n1} = \sigma$

If the charge sheet is not present then

$D_{n2} - D_{n1} = 0$

$D_{n2} = D_{n1}$ (5)

This is known as second boundary condition.

$\cos\theta_1 = \frac{D_{n1}}{D_1}$

$D_{n1} = D_1\cos\theta_1$ (6)

$\cos\theta_2 = \frac{D_{n2}}{D_2}$

$D_{n2} = D_2\cos\theta_2$ (7)

$D_1\cos\theta_1 = D_2\cos\theta_2$ (8)

$D_1 = \varepsilon_0 \varepsilon_{r1} E_1$

$D_2 = \varepsilon_0 \varepsilon_{r2} E_2$

$$\frac{\tan\theta_2}{\tan\theta_1} = \frac{\varepsilon_{r2}}{\varepsilon_{r1}}$$

θ_1 is angle of emergence.

θ_2 is angle of incidence.

This is relation between two dielectric surfaces.

UNIT - IV

MAGNETOSTATICS

Objectives:

- To introduce the students to Basic concepts of magnetic field, magnetic flux density.
- To introduce the basics of biot savart's law.
- To introduce the maxwell's second and third equation.

Syllabus:

Static magnetic fields – Biot-Savart's law – Oesterd's experiment - Magnetic field intensity (MFI) – MFI due to a straight current carrying filament – MFI due to circular, square and solenoid current – Carrying wire – Relation between magnetic flux, magnetic flux density and MFI – Maxwell's second Equation, div(B)=0.

Outcomes:

Students will be able to

- determine the magnetic field using biot savart's law and ampere's law.
- define magnetic field intensity or magnetic field strength (H) and derive expression for magnetic field for current filament, circular current loop polygon etc
- determine the H due to current sheet.
- Able to obtain maxwell's second equation from biot savart's law and third equation from ampere's lawevaluate the electric field from potential(Potential gradient)

UNIT IV

MAGNETOSTATICS

Steady current (or) D.C current (or) Time invariant current:

The motion of charges is at a constant rate with a time is called steady current. Magneto statics deals with magnetic field produced by steady current.

Magnetic field:

A static magnetic field can be produced from a permanent magnetic (or) a current carrying conductor. A steady current flowing in a straight conductor produces a magnetic field around it. The field exists as concentric circles having centers at the axis of the conductor.

If we hold the current carrying conductor by the right hand so that the thumb points the direction of current flow, the fingers point the direction of magnetic field. The unit of magnetic flux is Weber.

$$1 \text{ Wb} = 10^8 \text{ maxwells}$$

Magnetic flux density (B):

The magnetic flux per unit area is called magnetic flux density. The unit of magnetic flux density is Tesla (or) Wb/m^2.

The magnitude and direction of magnetic flux density due to current carrying conductor is given by Biot-Savart's law.

$$B = \frac{d\phi}{ds}$$

$$d\phi = B.ds$$

$$\phi = \int_s B.ds$$

Magnetic field intensity:

The magnetic field intensity at any point is the force experienced by a unit north pole of one weber strength when placed at that point. Unit is N/Wb, A/m (or) AT/m. It is denoted by $\overline{H}$.

Magneto Motive Force (mmf):

Magneto motive force is produced when an electric current flows through a coil of several turns. The Magneto motive force depends on the current and number of turns. Magneto motive force produces flux in a magnetic circuit. The unit of Magneto motive force is Ampere-turns.

Reluctance (s):

Reluctance is defined as the ratio of Magneto motive force to the flux produced. Reluctance is similar to the resistance in a electric circuit.

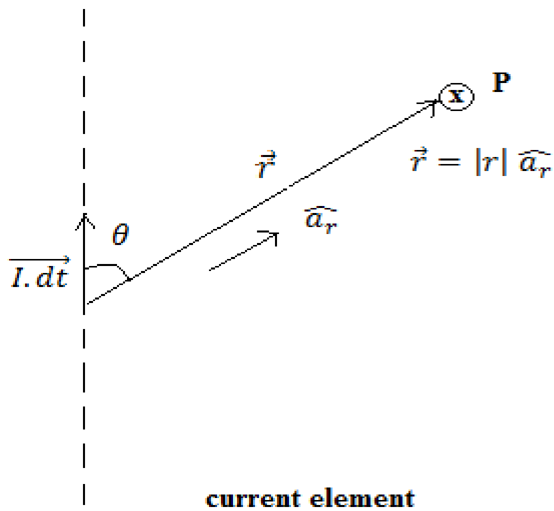
Reluctance is directly proportional to the length of the magnetic path and inversely proportional to the cross-sectional area of the path. The reciprocal of reluctance is called Permeance.

$$\text{Magneto motive force} = \text{Reluctance X flux}$$

$$\text{Reluctance} = \frac{\text{mmf}}{\text{flux}}$$

$$s = \frac{l}{\mu_0 \mu_r A}$$

Biot-Savart's law:



Steady current flowing through a straight conductor produces magnetic field in the form of concentric circles. The magnetic field intensity is given by Biot-Savart's law.

A straight conductor is assumed to be formed by several segments. Such segment is called current element. Current element is vector defined as $\overrightarrow{\mathrm{I.dl}}$.

Let the magnetic field intensity at P due to current element $\overrightarrow{\mathrm{I.dl}}$ be $\overrightarrow{\mathrm{dH}}$. The point P is at a distance 'r' m from the current element.

According to Biot-Savort's law, the magnitude of dH is

1) Directly proportional to the current element.
2) Inversely proportional to the square of the distance.
3) Directly proportional to the sine of the angle between current element and distance vector.

$\widehat{a_r}$ is the unit vector normal to the plane of the paper.

1) $|dH| \propto |I\,dl|$

2) $|dH| \propto \frac{1}{|r|^2}$

3) $|dH| \propto \sin\theta$

$$|dH| \propto \frac{|I\,dl|\sin\theta}{|r|^2}$$

Constant in M.K.S unit is $\frac{1}{4\pi}$

$$|dH| = \frac{|Idl|\sin\theta}{4\pi|r|^2}$$

$$\overrightarrow{dH} = |dH|\widehat{a_r}$$

$\overrightarrow{dH} = \frac{|I\,dl|\sin\theta}{4\pi|r|^2}\widehat{a_r}$ since $|r|\widehat{a_r} = \vec{r}$

$$\widehat{a_r} = \frac{\vec{r}}{|r|}$$

$$= \frac{|I\,dl|\sin\theta}{4\pi|r|^2}\frac{\vec{r}}{|r|}$$

$$= \frac{|I\,dl|\vec{r}\sin\theta}{4\pi|r|^3}$$

$$\overrightarrow{dH} = \frac{\overrightarrow{Idl}X\vec{r}}{4\pi|r|^3}$$

H due to entire conductor can be obtained by integration.

$$\vec{H} = \int \frac{\overrightarrow{I\,dl}X\vec{r}}{4\pi|r|^3}$$

$$\vec{H} = \frac{\int \overrightarrow{I\,dl}X\vec{r}}{4\pi|r|^3}$$

$$\vec{H} = \frac{\vec{Il}X\vec{r}}{4\pi|r|^3}$$

$\vec{B} = \mu H$ where μ = permeability

$$\mu = \mu_0\mu_r$$

μ_0 = permeability of free space

μ_r = relative permeability

$$\vec{B} = \frac{\mu}{4\pi|r|^3} SI\,\overrightarrow{dl}X\vec{r}$$

$$\vec{B} = \frac{\mu_0\mu_r}{4\pi}\frac{(\vec{Il}X\vec{r})}{|r|^3}$$

For air(or) free space $\mu_r = 1$

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{(\vec{Il}X\vec{r})}{|r|^3}$$

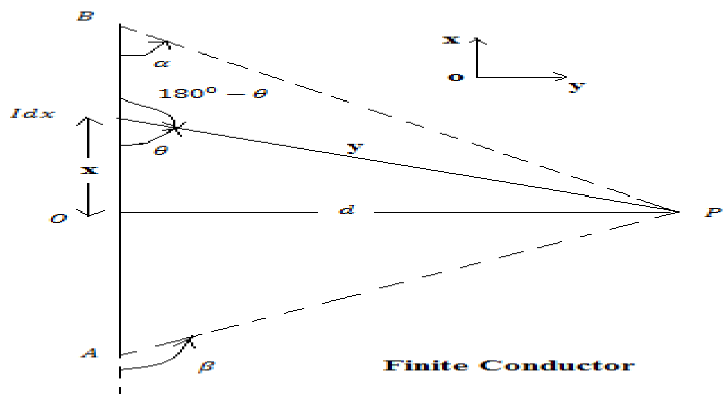
Since $\mu_0 = 4\pi X10^{-7}$ Henry/m

$\vec{B} = 10^{-7}\frac{(\vec{Il}x\vec{r})}{|r|^3}$ Wb/m^2

H due to finite conductor and infinite conductor:

We have to determine H due to a finite current carrying conductor at P. P is at a distance 'd' m from the origin.

Consider a current element Idx at a distance 'x' m from the origin. The distance vector between current element vector and point P is the vector $\vec{r}$.



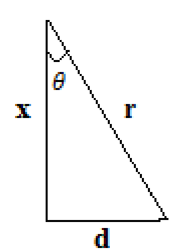
Let dH be the field intensity due to current element Idx which is at a distance 'x' m from the origin. By Biot-Savart's law,

$$\vec{dH} = \frac{\vec{Idl}X\vec{r}}{4\pi|r|^3}$$

$$|dH| = \frac{Idx|r|\sin(\pi - \theta)}{4\pi|r|^3}$$

$$dH = \frac{Idx\sin\theta}{4\pi|r|^2}$$

FromΔle , (1)



$\tan\theta = \frac{d}{x}$

$$x = d\cot\theta$$

$dx = -d\,cosec^2\theta$ (2)

$sin\theta = \frac{d}{r}$

$r = d\,cosec\theta$ (3)

From (1),(2) and (3)

$$dH = \frac{-I d\,cosec^2\theta d\theta sin\theta}{4\pi d^2 cosec^2\theta}$$

$$dH = -\frac{I}{4\pi d} sin\theta d\theta$$

Total magnetic field strength is obtained by integration,

$$H = \int_{\beta}^{\alpha} -\frac{I}{4\pi d} sin\theta d\theta$$

$$= -\frac{I}{4\pi d}\int_{\beta}^{\alpha} sin\theta d\theta$$

$$= \frac{I}{4\pi d}(cos\alpha - cos\beta)$$

$$\vec{H} = |H|\widehat{a_r}$$

$$\vec{H} = \frac{I}{4\pi d}(cos\alpha - cos\beta)\widehat{a_r}$$

As length tends to $\infty, \propto \rightarrow 0, \beta \rightarrow 180^0$

$$|H| = \frac{I}{4\pi d}(cos0 - cos180^0)$$

$$= \frac{I}{4\pi d}(1 - (-1))$$

$|H| = \frac{I}{2\pi d}$ A/m

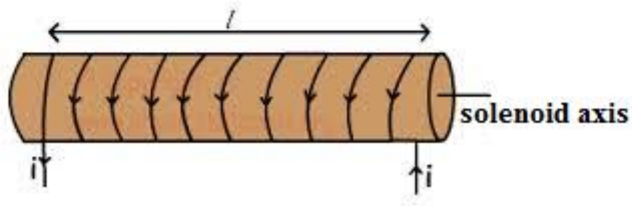
$\vec{H} = \frac{I}{2\pi d}\widehat{a_r}$ A/m

From this equation,it can be seen that the magnetic field intensity is inversely proportional to the distance.

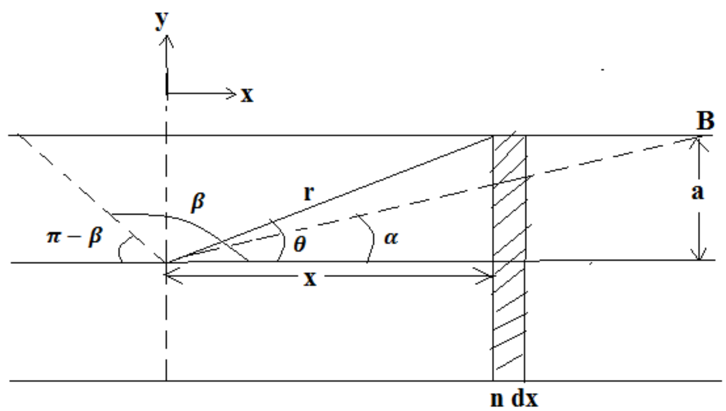
$\vec{B} = \frac{\mu}{2\pi}\frac{I}{d}\widehat{a_r}$ Wb/m^2

Solenoid:

A solenoid is a cylindrically shaped coil consisting of a large number of closely spaced turns of insulated wire wound usually on a non-magnetic frame.



H inside a solenoid: (Finite & infinite)



A solenoid has N no of turns on an iron rod. The turns can be assumed to be circular current loops. Considering a small section of length dx at a distance 'x' m from the origin . The no of turns in this section are ndx.

Let dH be the magnetic field intensity due to this section.

We know that $dH = \frac{Ia^2}{2r^3} ndx$ (1)

$$\sin\theta = \frac{a}{r}$$

$r = \frac{a}{\sin\theta} = a\text{cosec}\theta$ (2)

$$\cot\theta = \frac{x}{a}$$

$$x = a\cot\theta$$

$dx = -a\text{cosec}^2\theta d\theta$ (3)

Sub (2) and (3) in (1),

$$dH = -\frac{Ia^2}{2a^3 \text{cosec}^3\theta} nXa\text{cosec}^2\theta d\theta$$

$$= \frac{-In}{2}\frac{1}{\text{cosec}\theta}$$

$$= \frac{-In}{2}\sin\theta d\theta$$

Total magnetic field intensity can be obtained by varying θ from β to α.

$$H = \int_{\beta}^{\alpha} \frac{-In}{2}\sin\theta d\theta$$

$$= \frac{-In}{2}\int_{\beta}^{\alpha} \sin\theta d\theta$$

$$= \frac{In}{2}[\cos\alpha - \cos\beta]$$

Case 1: Let ‘P’ be the midpoint, $\beta = \pi - \alpha$

$$H = \frac{In}{2}[\cos\alpha - \cos(\pi - \alpha)]$$

$$= nI\cos\alpha$$

$$= \frac{NI}{L}\cos\alpha$$

$$H = \frac{NI}{L}\cos\alpha$$

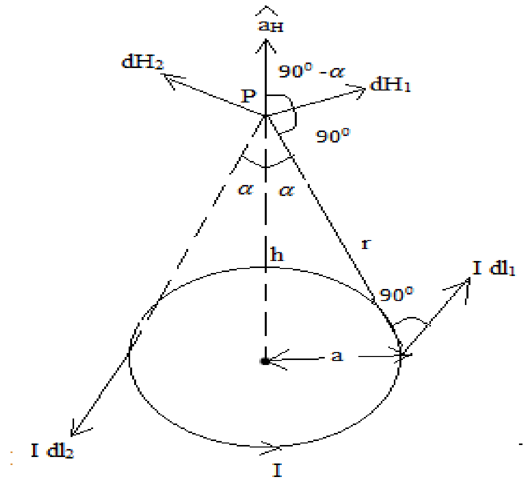
Case 2: For infinitely long solenoid, $\alpha \to 0, \beta = x$

$$H = \frac{In}{2}[\cos 0 - \cos\pi]$$

$$= nI$$

$H = \frac{NI}{L}$ AT/m

H due to a circular current loop:



We have to find H at point 'P' which is at a distance 'h' m from the center of the current loop. The circular loop can be divided into no of current elements. dH_1 and dH_2 are field intensities due to the elements Idl_1 and Idl_2 respectively. Similarly several vectors can be drawn due to several current elements. When these vectors are resolved, radial components get cancelled and normal components get added. There the direction of resultant magnetic field intensity is normal to the plane of the current loop. The same can be obtained using thumb rule (or) cork-screw rule to the current loop.

Normal component due to dH_1 is $dH_1 \cos(90-\alpha)$ i.e. $dH_1 \sin\alpha$.

Normal component due to dH_2 is $dH_2 \cos(90-\alpha)$ i.e. $dH_2 \sin\alpha$.

Therefore, sum of normal components would be resultant H_n.

$$H_n = dH_1 \sin\alpha + dH_2 \sin\alpha + \cdots \ldots + dH_n \sin\alpha$$

$$H_n = \int dH_1 \sin\alpha$$

$$H_n = \int \frac{\overrightarrow{Idl} X \vec{r}}{4\pi|r|^3} \sin\alpha$$

$$= \int \frac{|Idl||r|\sin\theta}{4\pi|r|^3} \sin\alpha$$

$$= \int \frac{|Idl||r| \ (1)}{4\pi|r|^3} \sin\alpha$$

$$= \int \frac{Idl}{4\pi|r|^2}\left(\frac{a}{r}\right)$$

$$= \frac{Ia}{4\pi|r|^3} X\, 2\pi a$$

$$= \frac{Ia^2}{2|r|^3}$$

$$|H| = \frac{Ia^2}{2|r|^3} A/m$$

$$\vec{H} = |H|\widehat{a_n}$$

$H = \frac{Ia^2}{2(a^2+h^2)^{\frac{3}{2}}}\widehat{a_n}$ A/m

Magnetic field intensity at the center of the current loop:

At the center, h=0.

$$H = \frac{Ia^2}{2a^3} == \frac{I}{2a}\widehat{a_n}\ A/m$$

If there are N no of turns,

$$\vec{H} = \frac{I\, N\, a^2}{2(a^2+h^2)^{\frac{3}{2}}}\widehat{a_n}\ AT/m$$

Maxwell's second equation:

From Biot-Savart's law, we know that

$$\vec{B} = \frac{\mu}{4\pi|r|^3}\left(\overrightarrow{I\, l}\, X\, \vec{r}\right)$$

Taking divergence on both sides,

$$\text{Div } B = \text{Div}\left(\frac{\mu}{4\pi|r|^3}\right)\left(\overrightarrow{I\, l}\, X\, \vec{r}\right)$$

$= \frac{\mu}{4\pi|r|^3}\text{Div}\left(\overrightarrow{I\, l}\, X\, \vec{r}\right)$ (1)

We know that $\text{Div}(\vec{u}\, X\, \vec{v}) = v.\text{curl } u - u.\text{curl } v$ (2)

Using (2), we can write (1) as

$$\text{Div } B = \frac{\mu}{4\pi r^3}(r.\text{curl } I\, l - I\, l.\text{curl } r)$$

Curl deals with rotation. The current element vector and distance vector have no rotation. Therefore curl $\overrightarrow{\mathrm{I\,dl}}$ and curl $\vec{r}$ vanishes.

$$\text{Div B} = \frac{\mu}{4\pi|r|^3}[0-0]$$

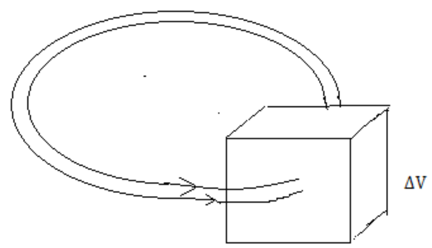
Therefore, Div B= 0.

This equation is known as field form (or) Differential form (or) Vector form of Biot-Savart's law.

This is also known as Maxwell's second equation.

Alternate proof for Div B=0:

Consider an infinitesimal volume ΔV as shown in fig. From fig, it can be seen that the flux entering and leaving are equal.



Net out flow of flux per unit volume is zero.

$$\emptyset = 0$$

$\int_s \mathrm{B.ds} = 0$ (1)

From divergence theorem,

$\int_s \mathrm{B.ds} = \int_v \mathrm{Div\,B.dv}$ (2)

From (1) and (2)

$$\int_s \mathrm{B.ds} = \int_v \mathrm{Div\,B.dv} = 0$$

$$\int_v \mathrm{Div\,B.dv} = 0$$

Div B=0

Therefore B is a solenoidal field. In electrostatics, positive charge acts as a source and negative charge acts as sink. The flux lines start from positive charge and terminate on the negative charge. Electric lines of flux are discontinues.

Magnetic lines of flux start at one point and terminate at the same point. These are continuous. This is nothing like a source and sink. Therefore isolated poles do not exist.

UNIT - V

FORCE IN MAGNETIC FIELDS

Objectives:

- To study the magnetic force and torque through Lorentz force equation in magnetic field environment like conductors and other current loops.

Syllabus:

Magnetic force - Moving charges in a Magnetic field - Lorentz force equation - force on a current element in a magnetic field - Force on a straight and a long current carrying conductor in a magnetic field - Force between two straight long and parallel current carrying conductors - Magnetic dipole and dipole moment - a differential current loop as a magnetic dipole - Torque on a current loop placed in a magnetic field.

Outcomes:

Students will be able to

- Define the concept of magnetic force
- Describe Lorentz force Equation
- Explain force on a current element in magnetic field
- Determine the force on straight and long current carrying conductor in a magnetic field
- Derive force between two straight long and parallel current carrying conductors in magnetic field
- List the applications of magnetic field force

UNIT - V

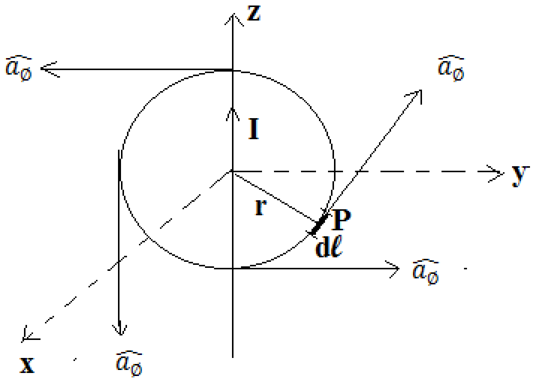
FORCE IN MAGNETIC FIELDS

Ampere's law (or) Ampere's circuital law:

The line integral of tangential component of M.F.I vector over a closed path is equal to current enclosed by that path (or) Work done by unit pole around a current carrying conductor is equal to current enclosed. If the conductor has 'N' no of turns,

1) $\oint H.dl = I_e$
2) $W.D = I_e = NI_e$
3) $\oint H.dl = NI_e$

Proof:



Consider a closed path around a current carrying conductor as shown in fig. The magnetic field at any point on the path is tangent. The point 'P' is at a distance 'r' from the conductor. Consider dl at point 'P' which is at direction $\widehat{a_\phi}$ is tangential to the circular path. From the Biot-Savart's law, the M.F.I along the conductor is given by

$$\bar{H} = \frac{I}{2\pi r}\widehat{a_\phi}$$

$$\bar{dl} = d_r\widehat{a_r} + rd\phi\widehat{a_\phi} + d_z\widehat{a_z}$$

$$\bar{dl} = rd\phi\widehat{a_\phi}$$

$$\oint H.dl = \oint \frac{I}{2\pi r}\widehat{a_\phi}.\ rd\phi\ \widehat{a_\phi}$$

$$\int_0^{2\pi} \frac{I}{2\pi}\,d\phi$$

$$= I$$

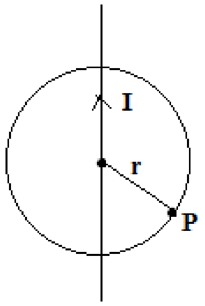
How to apply ampere's law:

It is used to determine the value of M.F.I (H) construct an ampere loop such that magnetic field is uniform and direction of magnetic field is tangential to the loop at every point. Then apply Ampere's law.

Applications:

1) H due to long conductor:

Construct an ampere loop with radius 'r' and apply Ampere's law.



$$W.D = I_e$$

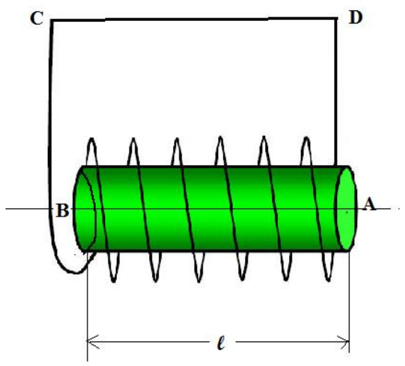
$$\oint H.dl = I_e$$

$$H.2\pi r = I$$

$$H = \frac{I}{2\pi r}\widehat{a_\phi}$$

Note: We cannot apply ampere's law to the finite conductor because the magnetic field is not uniform.

2) H due to a long solenoid:



A solenoid has 'N' turns and it carries a current 'I' A. The current direction is shown in fig. The magnetic field is towards left. The magnetic field outside the solenoid is zero.

Consider (or) construct rectangular loop (ABCDA) and apply Ampere's law,

$$\oint H.dl = NI$$

$$\int_{AB} H.dl + \int_{BC} H.dl + \int_{CD} H.dl + \int_{DA} H.dl = NI$$

Work done along CD is '0' since it is outside the solenoid.

Work done along DA and BC are '0' because H and dl are perpendicular.

$$\int_{AB} H.dl = NI$$

$$Hl = NI$$

$$H = \frac{NI}{l} \ AT/m \text{Type equation here.}$$

We can't apply ampere's law for finite solenoid because magnetic field is not uniform.

Maxwell's third equation:

According to ampere's law, the line integral of magnetic field intensity vector over a closed path is equal to current enclosed by that path.

$$\oint H.dl = I \qquad (1)$$

We know that,

$$I = \int_s J.ds \qquad (2)$$

From eq (1) and (2),

$$\oint H.dl = \int_s J.ds \qquad (3)$$

This is known as integral form of Ampere's law.

From the Stoke's theorem,

We know that

$$\oint H.dl = \int_s \nabla \times H ds \qquad (4)$$

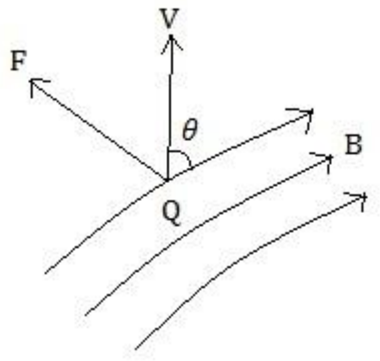
From (3) and (4),

$$\nabla \times H = J \qquad (5)$$

Where 'J' is known as point form (or) vector form (or) differential form of Ampere's law. This is also known as Maxwell's 3^{rd} equation.

Force on moving charge:

A charge particle is moving with velocity (v) is placed in magnetic field it experiences a force whose magnetic is proportional to the product of magnitudes of charge and velocity (**v**), flux density (**B**) and sine of angle between **B** and **v**.



F = Q v B sin θ.

$\mathbf{F} = Q\,(\mathbf{v}\times\mathbf{B})$

The direction of force is perpendicular to the both v and B and is given a unit vector in the direction of **v**×**B** **.**

$\mathbf{F} = Q\,(\mathbf{v}\times\mathbf{B})$

Lorentz's Force Equation :

A charged particle is moving with velocity is placed in magnetic field it experiences a force where magnitude is proportional to the product of magnitudes of charge and velocity (**v**) and flux density(**B**) and sine of angle between **B** and **v**.

F = Q v B sin θ.

$$\mathbf{F} = Q\,(\mathbf{v}\times\mathbf{B}) \qquad (4.1)$$

The direction of force is perpendicular to the both v and B and is given a unit vector in the direction of **v**×**B** **.**

When a charge Q is kept in an electric field it experiences a force.

$$\mathbf{F_2} = Q\,\mathbf{E} \qquad (4.2)$$

If the same charge is kept in an electro-magnetic field, the total force will be sum of $\mathbf{F_1}$ and $\mathbf{F_2}$ by superposition principle.

$\mathbf{F} = \mathbf{F_1} + \mathbf{F_2}$

$\mathbf{F} = Q\,(\mathbf{v}\times\mathbf{B}) + Q\,\mathbf{E}$

$$\mathbf{F} = Q\,[\,(\mathbf{v}\times\mathbf{B}) + \mathbf{E}\,] \qquad (4.3)$$

Which is known as Lorentz force equation

Force on a current element:

When a charged particle dQ is moving in a steady magnetic field it experiences a force due to different element.

d**F** = dQ (**v**×**B**)

d**F** = dQ **v B** sin θ $\mathbf{a_r}$

I=dQ/dt

dQ= I dt

d**F** = I dt vB sin θ $\mathbf{a_r}$

we know that dl = v dt

differential force d**F** = I dl B sin θ $\mathbf{a_r}$

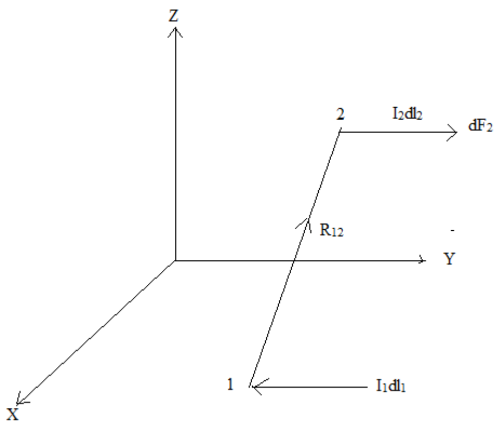
then total force on the current element **F** = I $\boldsymbol{l}$ × **B**

F = **- B** ×I$\boldsymbol{l}$

Where θ is angle between conductor and magnetic field.

Force between two differential current elements

Consider two differential current elements I_1dl_1, I_2dl_2 as shown in figure. According to Biot-Savarts have both the elements produce magnetic fields i.e. when a current (I_1) close through one of conductors, the magnetic field is developed around the conductors. If I_2 is placed in this magnetic field then force is exerted on the 2nd current element I_2dl_2.



The magnetic field at point (2) due to the current in the current element I_1dl_1 at point 1 is given by

$$d\boldsymbol{H}_1 = \frac{I_1 d\boldsymbol{l}_1 \times \mathbf{a}_{12}}{4\pi\, R^2{}_{12}} \qquad (4.4)$$

$$dB_1 = \frac{\mu\, I_1 d\boldsymbol{l}_1 \times \mathbf{a}_{12}}{4\pi\, R^2{}_{12}} \qquad (4.4)$$

The force on differential current element

$$d(d\mathbf{F}_2) = I_2 d\boldsymbol{l}_2 \times d\mathbf{B}_1$$

$$d(d\mathbf{F}_2) = I_2 d\boldsymbol{l}_2 \times \frac{\mu\, I_1 d\boldsymbol{l}_1 \times \mathbf{a}_{12}}{4\pi\, R^2{}_{12}}$$

$$d(d\mathbf{F}_2) = \frac{\mu\, I_1\, I_2}{4\pi\, R^2{}_{12}}\; d\boldsymbol{l}_2 \times (d\boldsymbol{l}_1 \times \mathbf{a}_{12})$$

The total force on conductor 2 due to current in conductor 1.

$$\mathbf{F}_2 = \int\int \frac{\mu\, I_1\, I_2}{4\pi\, R^2{}_{12}}\; d\boldsymbol{l}_2 \times (d\boldsymbol{l}_1 \times \mathbf{a}_{12})$$

This is also known as amperes torque equation. Similarly the total force on conductor 1 due to the current in conductor 2 is given by

$$\mathbf{F}_1 = \int\int \frac{\mu\, I_1\, I_2}{4\pi\, R^2{}_{12}}\; d\boldsymbol{l}_1 \times (d\boldsymbol{l}_2 \times \mathbf{a}_{21})$$

According to newtons 3rd law of motion, action and reaction are equal and opposite.

Force on a closed current loop:

The force exerted on a current element in a magnetic field is given by

$$F = \int Idl\ X\ B$$

$$F = -\int B\ X\ Idl$$

$$F = -I\int B\ X\ dl$$

Assume that **B** is uniform throughout the field. The force is

$$F = -IB\ X\int dl$$

$$F = 0 \qquad since \int dl = 0$$

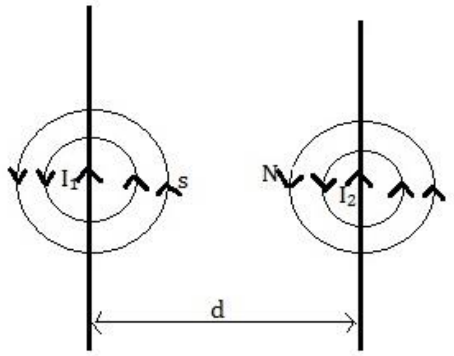
If a closed filamentary circuit is placed in a uniform magnetic field, it does not experience a force.

If magnetic field is not uniform throughout the field, the force is not zero.

Force between two straight parallel current carrying conductors:

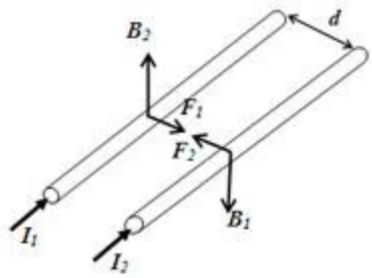
In figure both conductors carrying current in the same direction, the magnetic field of first conductor in the upward direction and magnetic field due to second conductor is in the downward direction. The upward direction of the magnetic field is represented by south pole and downward as north pole. There is a force of attraction between two conductors carrying current in same direction.

From figure there is a force of repulsion between two conductors carrying current in opposite directions.



Force between two straight parallel current carrying conductors in same direction:

Consider two straight long parallel conductors placed at a distance 'd' m apart. We have to determine the force between two conductors per meter length.



Consider conductor one produces magnetic field which is located at conductor two is given by

$$\boldsymbol{B}_1 = \frac{-\mu I_1}{2\pi d} \boldsymbol{a}_z \; wb/m^2$$

Conductor two carrying current I_2 and is placed in field produced by conductor one. Then, the conductor two experiences a force.

$\mathbf{F_2} = I\,(\boldsymbol{l} \times \mathbf{B})$

$\mathbf{F_2} = -I_2\,(\boldsymbol{l}\,\mathbf{a_x} \times -B_1\,\mathbf{a_z})$

$\mathbf{F_2} = -I_2\, l\, B_1\,\mathbf{a_y}$ N

The direction of force is along –ve y-axis similarly, the magnetic field produced by conductor two at conductor one is given by

$$\boldsymbol{B}_2 = \frac{\mu I_2}{2\pi d}\boldsymbol{a}_z \ wb/m^2$$

The force experienced by conductor one due to current in conductor two is given by

$\mathbf{F_2} = -I_1 (\boldsymbol{l}\ \mathbf{a_x} \times B\ \mathbf{a_z})$

$\mathbf{F_2} = -I_1\ l\ B\ \mathbf{a_y}$

Therefore ,

$\mathbf{F_2} = -I_2\ l \frac{-\mu I_1}{2\pi d}\ \mathbf{a_y}$

$\mathbf{F_2}/l = \frac{\mu I_1 I_2}{2\pi d}\ \mathbf{a_y}$

$\mathbf{F} = \frac{\mu I_1 I_2}{2\pi d}\ \mathbf{a_y}$ N/m

From this we can conclude that there is a force of attraction between two conductors carrying current in the same direction.

If $I_1 = I_2 = 1A$; $d = 1m$; $\mu = \mu_0$

$F = \frac{\mu_0}{2\pi} = 2 \times 10^{-7}\ N/m$

1A is defined as the value of current which is flowing in two infinitively long conductors with their centres in a part in free space produce a force between them is 2×10^{-7} N/m.

Magnetic torque:

The force on a filamentary closed circuit is given by

$$F = -I\int B \times dL$$

and assume a uniform magnetic flux density, then B may be removed from the integral:

$$F = -IB \times \int dL$$

However, In closed line integrals in an electrostatic potential field dL = 0, and therefore the force on a closed filamentary circuit in a uniform magnetic field is zero.

If the field is not uniform, the total force need not be zero.

This result for uniform fields does not have to be restricted to filamentary circuits only. The circuit may contain surface currents or volume current density as well. If the total current is divided into filaments, the force on each one is zero, as we showed above, and the total force is again zero. Therefore any real closed circuit carrying direct currents experiences a total vector force of zero in a uniform magnetic field.

Although the force is zero, the torque is generally not equal to zero.

In defining the torque, or moment, of a force, it is necessary to consider both an origin at or about which the torque is to be calculated, as well as the point at which the force is applied. In Fig. 9.5a, we apply a force F at point P, and we

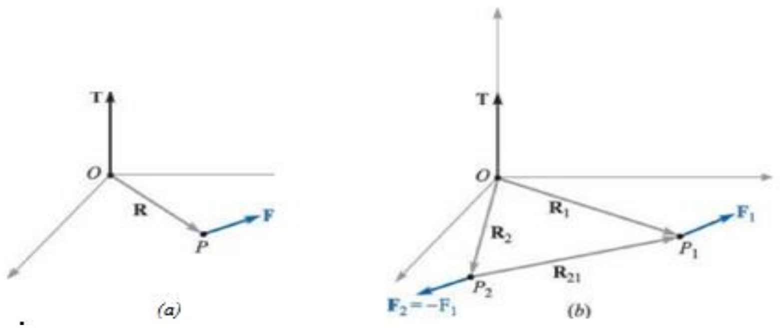


FIGURE 9.5

(a) Given a lever arm R extending from an origin 0 to a point P where force F is applied, the torque about 0 is T = R x F. (h) If F_2 = -F_1, then the torque T = R_{21} x F_1 is independent of the choice of origin for R_1 and R_2.

Establish an origin at *O* with a rigid lever arm R extending from 0 to P. The torque about point *O* is a vector whose magnitude is the product of the magnitudes of R, of F, and of the sine of the angle between these two vectors. The direction of the vector torque T is normal to both the force F and lever arm R and is in the direction of progress of a right-handed screw as the lever arm is rotated into the force vector through the smaller angle. The torque is expressible as a cross product,

$$T=RxF$$

Now let us assume that two forces, F_1 at P_1 and F_2 at P_2, having lever arms R_1 and R_2 extending from a common origin 0, as shown in Fig. 9.5b, are applied to an object of fixed shape and that the object does not undergo any translation. Then the torque about the origin is

$T = R_1 \text{ x } F_1 + R_2 \text{ X } F_2$

where

$F_1 + F_2 = 0$

and therefore

$T = (R_1 — R_2) \text{ x } F_1 = R_{21} \text{ x } F_1$

The vector $R_{21} = R_1 - R_2$ joins the point of application of F_2 to that of F_1 and is independent of the choice of origin for the two vectors R_1 and R_2. Therefore, the torque is also independent of the choice of origin, provided that the total force is zero. This may be extended to any number of forces.

Consider the application of a vertically upward force at the end of a horizontal crank handle on an elderly automobile. This cannot be the only applied force, for if it were, the entire handle would be accelerated in an upward direction. A second force, equal in magnitude to that exerted at the end of the handle, is applied in a downward direction by the bearing surface at the axis of rotation. For a 40-N force on a crank handle 0.3 m in length, the torque is 12 Nm. This figure is obtained regardless of whether the origin is considered to be on the axis of rotation (leading to 12 Nm plus 0 m), at the midpoint of the handle (leading to 6 Nm plus 6 m), or at some point not even on the handle or an extension of the handle.

We may therefore choose the most convenient origin, and this is usually on the axis of rotation and in the plane containing the applied forces if the several forces are coplanar.

With this introduction to the concept of torque, let us now consider the torque on a differential current loop in a magnetic field B. The loop lies in the xy plane (Fig. 9.6); the sides of the loop are parallel to the x and y axes and are of length dx and dy. The value of the magnetic field at the center of the loop is taken as Bo. Since the loop is of differential size, the value of B at all points on the loop may be taken as Bo. (Why was this not possible in the discussion of curl and

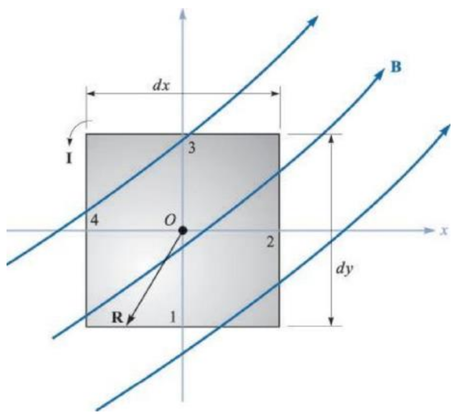


Fig: A differential current loop in a magnetic field B. The torque on the loop is $dT = I(dxdya_z) \times B_o = I\, dS \times B$

divergence?) The total force on the loop is therefore zero, and we are free to choose the origin for the torque at the center of the loop.

The vector force on side 1 is

$dF_1 = I\, dx\, a_x \times B_o$

or

$dF1 = I\, dx(B_{oy}a_z, — B_{oz}a_y)$

For this side of the loop the lever arm R extends from the origin to the midpoint of the side, $R_1 = -0.5dy\, a_y$, and the contribution to the total torque is

$dT_1 = R_1 \times dF_1$

$= -0.5dy\ a_y \times I\ dx(B_{oy}a_z — B_{oz}a_y)$

$= -0.5dx\ dy\ IB_{oy}a_x$

The torque contribution on side 3 is found to be the same,

$dT_3 = R_3 \times dF_3 = -0.5dy\ a_y \times (—I\ dx\ a_x \times B_o) = -0.5dx\ dy\ IB_{oy}a_x = dT_1$

and

$dT_1 + dT_3 = —dx\ dy\ IB_{oy}a_x$

Evaluating the torque on sides 2 and 4, we find

$dT_2 + dT_4 = dx\ dy\ IB_{ox}a_y$

and the total torque is then

$dT = I\ dx\ dy(B_{ox}a_y — B_{oy}a_x)$

The quantity within the parentheses may be represented by a cross product,

$$dT = I\ dx\ dy(a_z \times B_o)$$

or

$$dT = I\ dS \times B \qquad (15)$$

where dS is the vector area of the differential current loop and the subscript on Bo has been dropped.

We now define the product of the loop current and the vector area of the loop as the differential magnetic dipole moment dm, with units of Am^2. Thus

$$dm = IdS \qquad (16)$$

and $$dT = dm \times B \qquad (17)$$

If we extend the results we obtained differential electric dipole by determining the torque produced on it by an electric field, we see a similar result,

$$dT = dp \times E$$

Equations (15) and (17) are general results which hold for differential loops of any shape, not just rectangular ones. The torque on a circular or triangular loop is also given in terms of the vector surface or the moment by (15) or (17).

Since we selected a differential current loop so that we might assume B was constant throughout it, it follows that the torque on a planar loop of any size or shape in a uniform magnetic field is given by the same expression,

$$T = IS \times B = m \times B \quad (18)$$

We should note that the torque on the current loop always tends to turn the loop so as to align the magnetic field produced by the loop with the applied magnetic field that is causing the torque. This is perhaps the easiest way to determine the direction of the torque.

Magnetic dipole:

A circular loop of small area is called magnetic dipole.

Magnetic dipole moment:

It is the product of current and area.

$\mathbf{m} = I\, A\, \mathbf{a_n}$ A-m^2

If the loop has 'N' no. of turns then

$\mathbf{m} = NI\, A\, \mathbf{a_n}$ AT-m^2

where $\mathbf{a_n}$ is the unit vector normal to the plane of the loop.

Consider a magnetic material having 'n' no. of dipoles then the total magnetic dipole moment,

$\mathbf{m} = \mathbf{m_1} + \mathbf{m_2} + \ldots\ldots\ldots\ldots + \mathbf{m_n}$

Magnetic polarisation or magnetisation:

It is ratio of magnetic dipole moment per unit volume.

$$\boldsymbol{M} = \lim_{\Delta v \to 0} \frac{\boldsymbol{m}}{\Delta v}$$

$$\boldsymbol{M} = \frac{\boldsymbol{m}}{v} \quad \frac{A}{m} or \ AT/m$$

UNIT – VI

TIME VARYING FIELDS

Objectives:

- To study time varying and Maxwell's equations in different forms andMaxwell's fourth equation for the induced Emf.

Syllabus:

Time varying fields – Faraday's laws of electromagnetic induction – Itsintegral and point forms – Maxwell's fourth equation, Curl (E)=-∂B/∂t –Statically and Dynamically induced EMFs – Simple problems -Modificationof Maxwell's equations for time varying fields – Displacement current –Poynting Theorem and Poynting vector.

Outcomes:

Students will be able to

- Define the function of time varying fields
- Derive faraday's law of electromagnetic induction
- Identify the application of faraday's law of electromagnetic induction
- List applications of faraday's law in the field of electrical engineering

UNIT – VI

TIME VARYING FIELDS

Faraday's First law:

Whenever conductor cuts the flux (flux not constant) emf is induced in the conductor.

e = - $N\ \frac{d\phi}{dt}$

Faraday's second law:

The emf induced per turn is negative of the rate of change of flux linkages.

e = - $\frac{d\phi}{dt}$

'-' sign is an indication that the emf is in such direction has to produce a current whose flow is added to original flow to reduce magnitude of emf.

Lenz's Law:

Consider a wire loop shown with a bar magnet moving upwards show that the flux through the loop increasing, this results an induced current in the loop flowing in a direction such that loop magnetic field oppose the motion of the magnet since like pole repel each other.

The magnet is moving down away from the flux through the loop is decreasing, this results an induced current flowing in a direction such that loop magnetic field oppose the motion of the magnet. Since the unlike poles attract eachother. Thus the induced current in the loop is always in such a direction to oppose the change produces it.

Let us suppose that emf in circuit b acts to send a current in the same direction as current in circuit a. Such current will strengthen original magnetic field sut up by increasing current in circuit a. The induced emf and the current, flux increases. This is impossible therefore direction of emf circulate a current in circuit b in such a direction as to oppose the increasing in the original speed.

According to Lenz's law the induced voltage acts to produce the opposing flux. Lenz's law state that the direction of induced emf is such as it tends to oppose the cause or more explicitely direction induced emf in any current produces or tends to oppose the change of flux that is produced in it.

Maxwell's Fourth Equation:

Consider a coil kept in the magnetic field as shown. X – indicates the flux is perpendicular to the plane of the coil. We know that

V = $\int E\ .dl$

From Faraday's law

e = - $N\frac{d\phi}{dt}$

= - $\frac{d\phi}{dt}$

$\phi = \int B \,.\, dS$

$\int E \,.\, dl$ = - $\int \frac{dB}{dt}$ dS

$\int E \,.\, dl = \int \nabla XE$ dS

∇XE = - $\frac{dB}{dt}$

Point form or differential form of Maxwell's 4th equation.

From the above equation it can be observed that time varying magnetic field produces Electric field.

Statically induced emf:

Whenever change in flux that is passing through the conductor produces the emf on the conductor. This is known as statically induced emf. Consider a coil kept in the magnetic field shown. X- indicates increase in flux is perpendicular to the plane of the coil. We know that potential difference is the integral of E.dl at no load.

V = $\int E \,.\, dl$

e = - $\frac{d\phi}{dt}$

At no load e = V

e = - $\frac{d\phi}{dt} = \int E \,.\, dl$

Dynamically Induced emf:

Whenever a conductor moves with a velocity passing through a stationary magnetic field. The emf is induced in the conductor. This emf is known as dynamically induced emf.

A magnetic field is established perpendicular to the plane of paper as shown. The circuit has two metal rails connected at the upper end of the galvanometer. Amovable conductor is arranged to slide along the rails and maintain contact with the rails. The conductor moves with velocity v and distance dx in dt seconds. The emf induced in the conductor since flux linked by it increased.

From Faraday's Law

e = - $\frac{d\phi}{dt}$

$= - \frac{d}{dt}$ (BS)

$= - B \frac{dS}{dt}$

$= -B \frac{l\,dx}{dt}$

e $= -Blv$

Faraday's disc G/r's:

A disc of radius a is attached to the shaft or axis, the system is supported by 2 bearings on either side. One brush is brushed on the shaft and other is brushed over the edge of the disc. A magnetic field is applied in a direction perpendicular to the plane of the disc. This system is driven by prime mover at a speed of N rpm. When the disc rotates it cuts the flux an emf is induced in the disc the emf is collected using brushes as shown. According to Faraday's second law the magnitude of emf is equal to time rate of change of flux. The disc is assumed to be formed by several meters of radius a. Consider a segment OA occupies position OB after dt seconds the area of triangle OAB is approximated to right angle triangle.

According to Faraday's law the magnitude of emf

$e = \frac{d\phi}{dt}$

$= \frac{d}{dt}$ (BS)

$= B\frac{dS}{dt}$

$= B\frac{d}{dt}(\frac{1}{2}a^2\, d\theta)$

$= \frac{1}{2}Ba^2\,\omega$

From the above equation it is seen that emf is directly proportional to B.

Modified Ampere's Law:

As per Ampere's law the line integral of magnetic field intensity vector over a closed path is equal to current enclosed by that path.

$\int H\,.\,dl = I$ (1)

We know that $I = \int J\,.\,ds$ (2)

$\int H\,.\,dl = \int J\,.\,ds$ (3)

By Stoke'stherom

$\int H\,.\,dl = \int \nabla X H\; ds$ (4)

From equations 3 & 4

$\nabla X H = J$ (5)

$\nabla . \nabla X H = \nabla . J$

$\nabla . J = 0$ (6)

From continuity equation

$\nabla . J = -\frac{d\rho}{dt}$ (7)

Equation 7 contradicts with equation 6, This is called Maxwell's dynamo equation.

Maxwell stated that basic ampere's law is not valid for time varying fields and valid for time invariant fields.

Maxwell has done following modification that the ampere's law is valid for time varying fields.

From the continuity equation

$\nabla . J = -\frac{d\rho}{dt}$

From the point form of Gauss law

$\nabla . D = \rho$ (8)

$\nabla . J = -\frac{d}{dt} (\nabla . D)$

$\nabla . J + \nabla . \frac{d}{dt} (D) = 0$

$\nabla . J + \nabla . J_d = 0$ (9)

Where $J_d = \frac{d}{dt} (D)$ is displacement current density.

We know that $\nabla . \nabla X H = 0$

$\nabla . J + \nabla . J_d = \nabla . \nabla X H$

$\nabla X H = J + J_d$

$\int \nabla X H \, dS = \int (J + J_d) \, dS$

$\int \nabla X H \, dS = I + I_d$ (10)

From Stokes theorem,

$\int H . dl = \int \nabla X H \, ds$

$\int H . dl = I + I_d$ (11)

According to modified Ampere's law the line integral of tangential component of magnetic field intensity vector for a closed path equal to I+ I_d.

Displacement current:

The current through a capacitor is known as displacement current.

$J_d = \frac{dD}{dt}$

$\frac{I_d}{A} = \frac{d\varepsilon E}{dt}$

$= \epsilon \frac{dE}{dt}$

$E = \frac{V}{d}$

$I_d = A\frac{\epsilon}{d}\frac{dV}{dt}$

$I_d = C\frac{dV}{dt}$

From above equation it can be seen that I_d is the current through capacitor. The current flowing when the electric field across the capacitor is increasing or decreasing. The motion of the slider stop then the motion of the charges stopped and ammeter reading is zero. I_d flows when the electric field is change with time. Consider a sinusoidal voltage source connected to parallel plate capacitor. Construct 2 blocks as shown. The second surface is constructed such that it encloses top capacitor plate alone no conduction current flows through it since it lies in dielectric.

Apply Ampere's law to the surface S_1 and S_2

$\int H \,.\, dl = I_1$

$\int H \,.\, dl = I_d$

A free charge being stored and removed each capacitor plate a time varying field is produced between plates. This time varying field produces the I_d.

Conduction current and Displacement current:

From the point form of Ohm's law conduction current density,

$J = \sigma E$

$J = \sigma\, E_{max} \sin\omega t$

$\frac{I}{A} = \sigma \frac{V_{max}}{d} \sin\omega t$

$I = A\sigma \frac{V_{max}}{d} \sin\omega t$

$I = I_{max}\sin\omega t$

$I_{max} = A\sigma\frac{V_{max}}{d}$

$I_{rms} = A\sigma\frac{V_{max}}{\sqrt{2}d}$

$J_d = \frac{dD}{dt}$

$J_d = \frac{d(\epsilon E)}{dt}$

$J_d = \epsilon\frac{d(E_{max}\sin\omega t)}{dt}$

$J_d = \epsilon E_{max}\omega cos\omega t$

$I_d = A\epsilon\frac{V_{max}}{d}\omega cos\omega t$

$I_d = I_{dmax}\cos\omega t$

$I_{dmax} = A\epsilon\frac{V_{max}}{d}\omega$

$I_{drms} = A\epsilon\frac{V_{max}}{\sqrt{2}d}\omega$

I_d is directly proportional to the frequency. Conduction current follows sine law and displacement current follows cosine law.

Differences between conduction and displacement current

1. Conduction current obeys ohm's law as i= V/Rbut displacement current does not obey ohm's law.
2. Conduction current density is represented by $J_c=\sigma E$whereas displacement currentdensity is given by $J_d = \frac{dD}{dt} = \varepsilon\frac{dE}{dt}$.
3. Conduction current is the actual current whereas displacement current is the apparentcurrent produced by time varying electric field.

Pointing Vector:

It is defined as the cross product of the vectors E & H.

S = EXH

= EH $\sin\theta$

$E = \frac{v}{l}$

H = $\frac{I}{2\pi r}$

S = $\frac{vi}{2\pi rl}$

S = $\frac{Power}{Area}$

It gives power/unit area and gives magnitude and direction in which power flows in time EMF. The direction of power flow at any point is normal to the E and H.

Poynting Theorem:

The energy generated per unit volume and second is equal to same of the energy stored in EMF per unit volume and second and energy crossed per unit volume and second.

We know that S = EXH

$\nabla.$ S = $\nabla.$ (EXH)

From vector identities

$\nabla.$ S = H. ∇XE - E.∇XH

$= H(-\frac{dB}{dt}) - E\,(J+\frac{dD}{dt})$

$= -\mu H\frac{dH}{dt}$ - EJ $- \epsilon E\ \frac{dE}{dt}$

$= -\frac{d}{dt}\,(\frac{1}{2}\,\mu H^2)$ – EJ - $\frac{d}{dt}\,(\frac{1}{2}\,\epsilon E^2)$

-EJ = $\nabla.$ S + $\frac{d}{dt}\,(\frac{1}{2}\mu H^2 + \frac{1}{2}\,\epsilon E^2)$

-$\int EJ\ dv$ = $\int \nabla.$ S dv + $\int \frac{d}{dt}\left(\frac{1}{2}\mu H^2 + \frac{1}{2}\epsilon E^2\right)$ dv

$\int \nabla.$ S dv = $\int$ S. dS

-$\int EJ\ dv$ - $\int \frac{d}{dt}\left(\frac{1}{2}\mu H^2 + \frac{1}{2}\epsilon E^2\right)$ dv = $\int \nabla.$ S dv = $\int$ S. dS

This is integral form of Poynting theorem.